Physics on a Branched Knotted Spacetime Manifold

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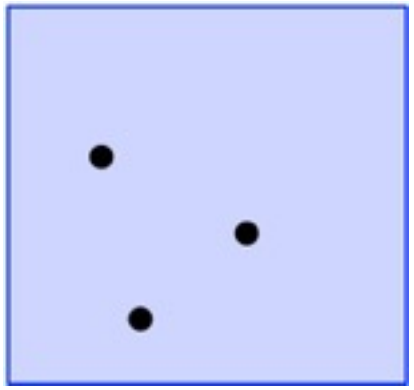
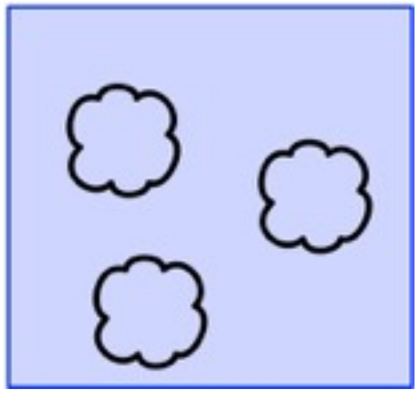
Brief historical context

General relativity:
Pointlike particles produce infinities.



M-theory:
Stringy particles maintain finiteness, but the diversity of Calabi-Yau spaces makes predictions difficult.



This theory:
Particles are knots in spacetime, never pointlike.

Overview

Manifold properties

embedding, branching, constraints, knotting

Quantum mechanics

particle amplitude, particle interference

Entropy, Lagrangian, and fields

entropy maximization, field equations, applications

Basic properties of the theory

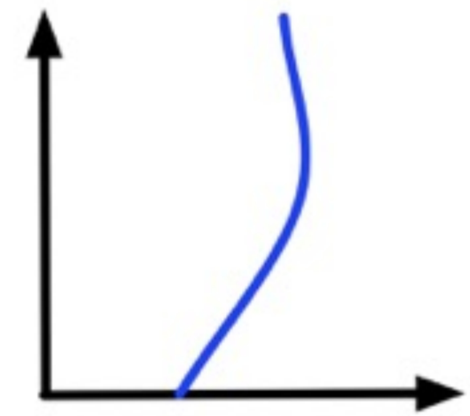
The spacetime manifold

M is the spacetime 3+1-manifold.
M is embedded in a Minkowski 5+1-space Ω.

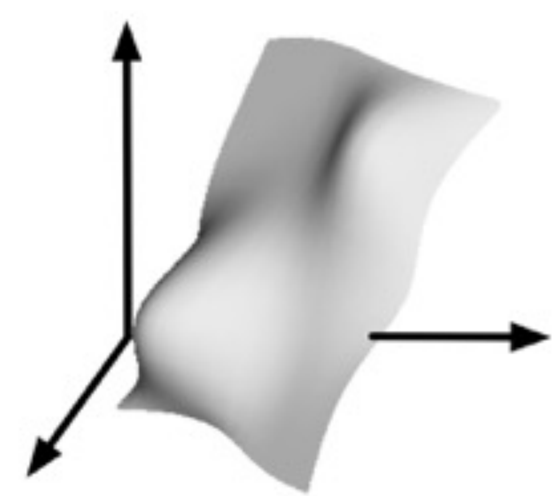
The coordinate on Ω is x^{ν}.
The metric on Ω is $\eta_{\mu\nu}$.
The metric restricted to M is $\bar{\eta}_{\mu\nu}$.

A manifold with n dimensions in an n+2-dimensional space can be knotted (knot theory).

Knots on M represent fermions.

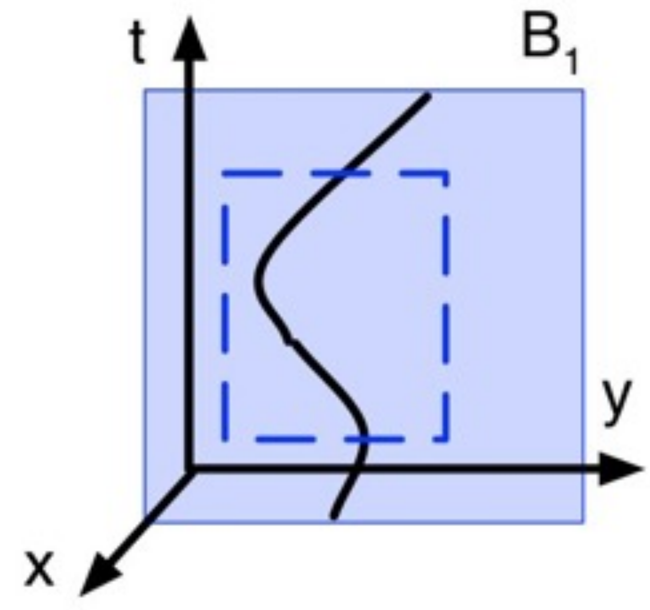


A 0+1 spacetime in a Minkowski 1+1-space



A 1+1 spacetime in a Minkowski 2+1-space

The spacetime manifold



In Ω, the manifold M branches and recombines.

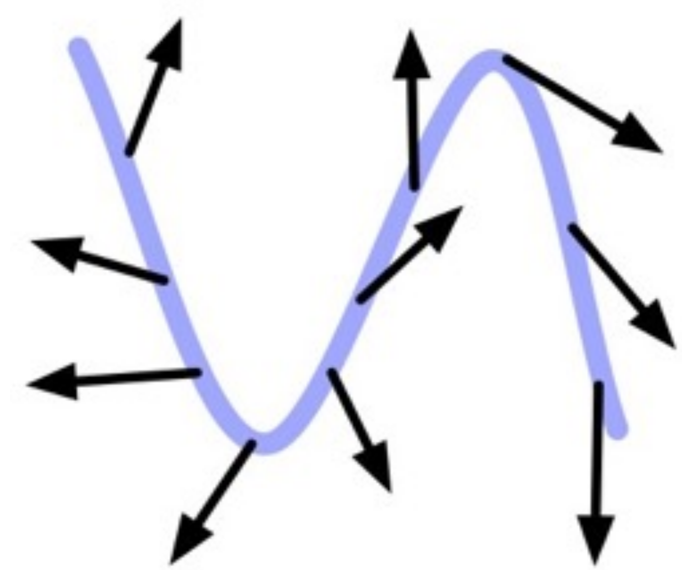
Knots on M also branch and recombine.

This results in interference and a probabilistic theory.

Conformal weight and field

The manifold M has a conformal weight ρ.

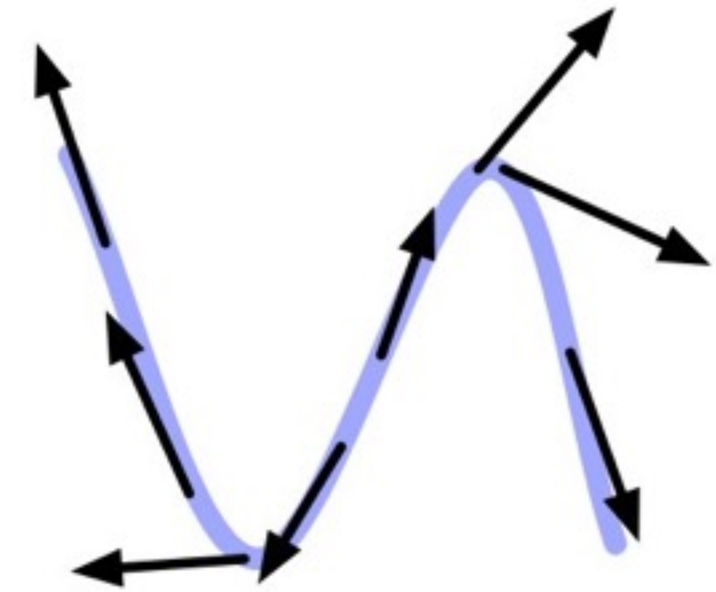
M also has a field A^{ν} that takes values in the tangent space of Ω.



We will generate a metric $g_{\mu\nu}$ on M by taking derivatives of A^{ν}.

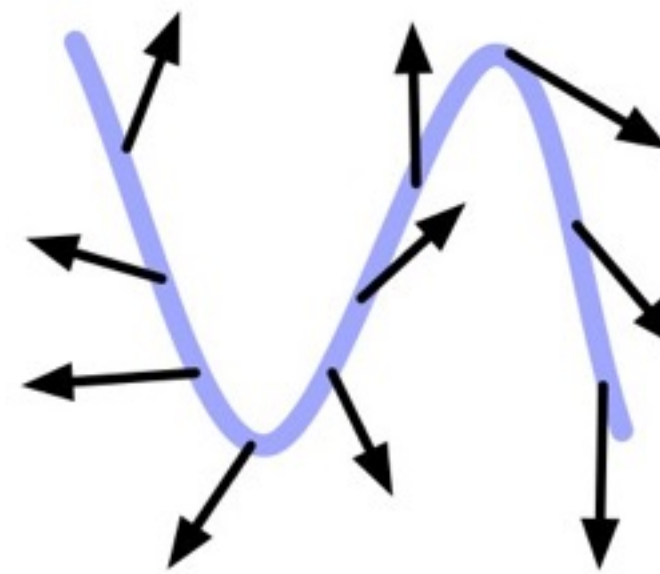
Background: Derivatives on embedded manifolds

For a tensor T in the tangent space of M (4-D spacetime), derivatives may be covariant (with parallel translation) or ordinary partials.



$$\nabla_\mu T \quad \partial_\mu T$$

For a tensor T not in the tangent space of M, derivatives are ordinary partials. Derivatives are w.r.t. the metric $\eta_{\mu\nu}$ of the space Ω (6-D).

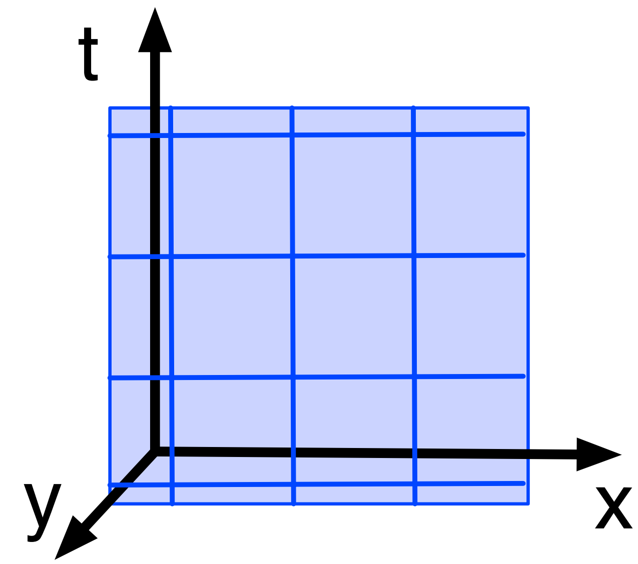


$$\partial_\mu T$$

In this theory nearly all derivatives are ordinary partials.

Metric

We restrict the A^ν vector field such that $\det(A_{\alpha,\mu}A^{\alpha}{}_{,\nu}) = -1$
(For example $A^\nu = x^\nu$ always works.)

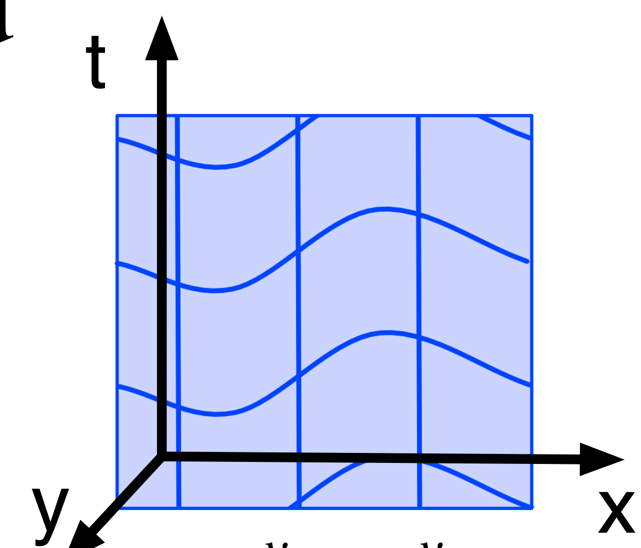


$A^\nu = x^\nu$

Then we make a metric $g_{\mu\nu}$ using the conformal weight ρ.

$$g_{\mu\nu} = \rho^2 A^{\alpha}{}_{,\mu} A_{\alpha,\nu}$$

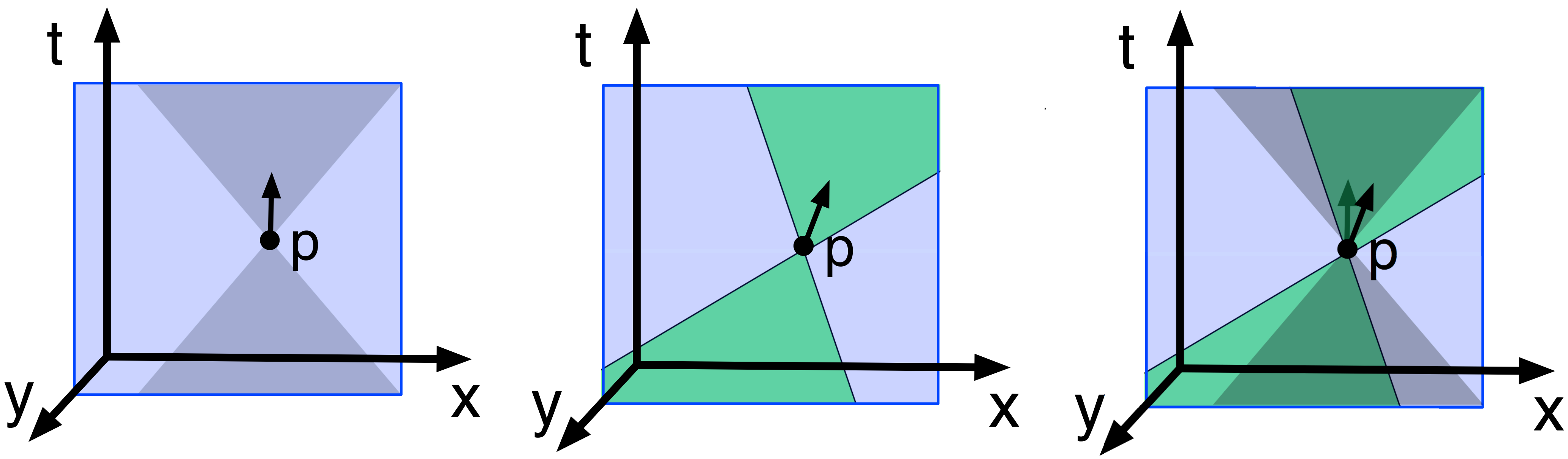
Locally, A^ν is analogous to a coordinate and $g_{\mu\nu}$ is the rate at which A^ν changes, weighted by ρ.



$A^\nu \neq x^\nu$

(If $A^\nu = x^\nu$ then $g_{\mu\nu} = \rho^2 \bar{\eta}_{\mu\nu}$.)

A^{ν} field



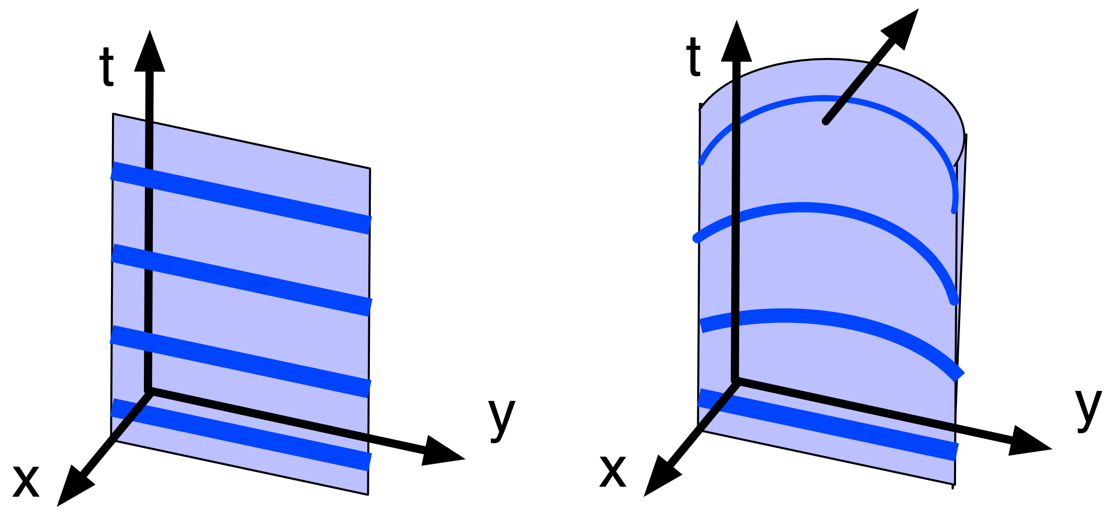
Call the future cone of $\eta_{\mu\nu}$: η^{+}

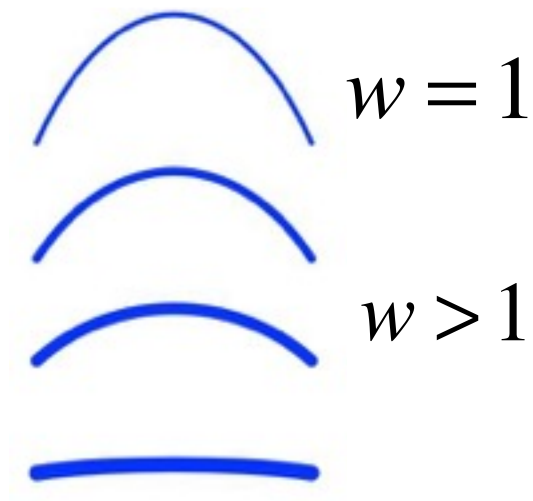
Call the future cone of $g_{\mu\nu}$: g^{+}

We require that the metric $g_{\mu\nu}$ be constrained such that, at every point p, g^{+} intersects η^{+}.

We will identify A^{ν} with the electromagnetic potential.

$$\hat{R}^{\mu\nu} = 0$$



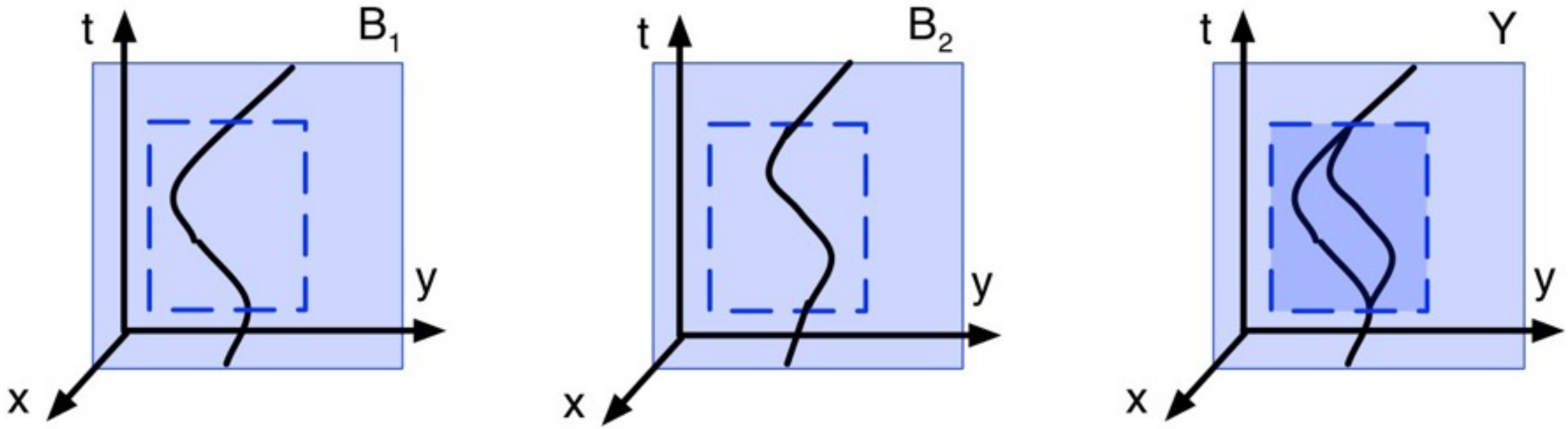


What prevents unbounded expansion into the co-dimension (x^4 and x^5)?

Use metric $g_{\mu\nu} = \rho^2 A^{\alpha}_{,\mu} A_{\alpha,\nu}$
Then require Ricci flatness w.r.t. $g_{\mu\nu}$: $\hat{R}^{\mu\nu} = 0$

Define a weight $w = \sqrt{|\det g|} = \rho^4$ and require $w \geq 1$.
Stretching the manifold reduces the weight w.
Stretching is limited by $w \geq 1$.

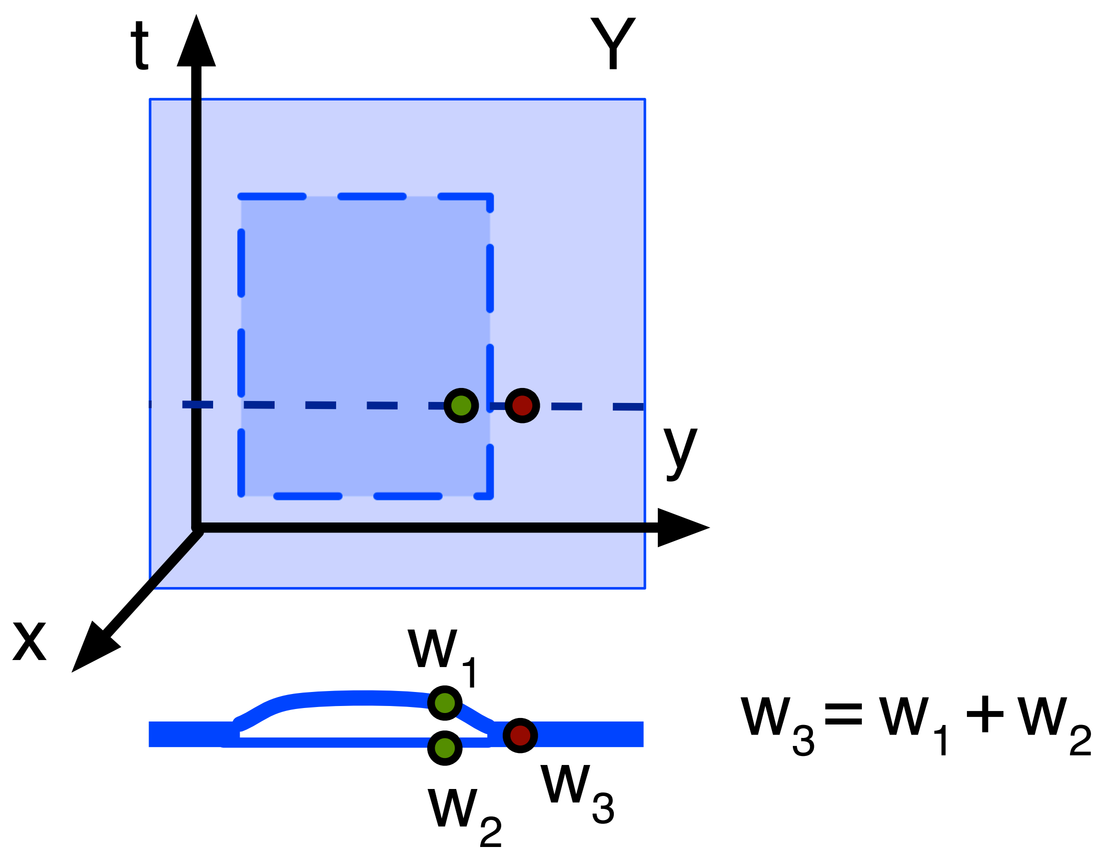
Quantum branches



To allow quantum mechanics, we let M be a branched manifold. The branches separate and recombine.

Knots on the branches also separate and recombine. Each Feynman diagram represents a class of branches.

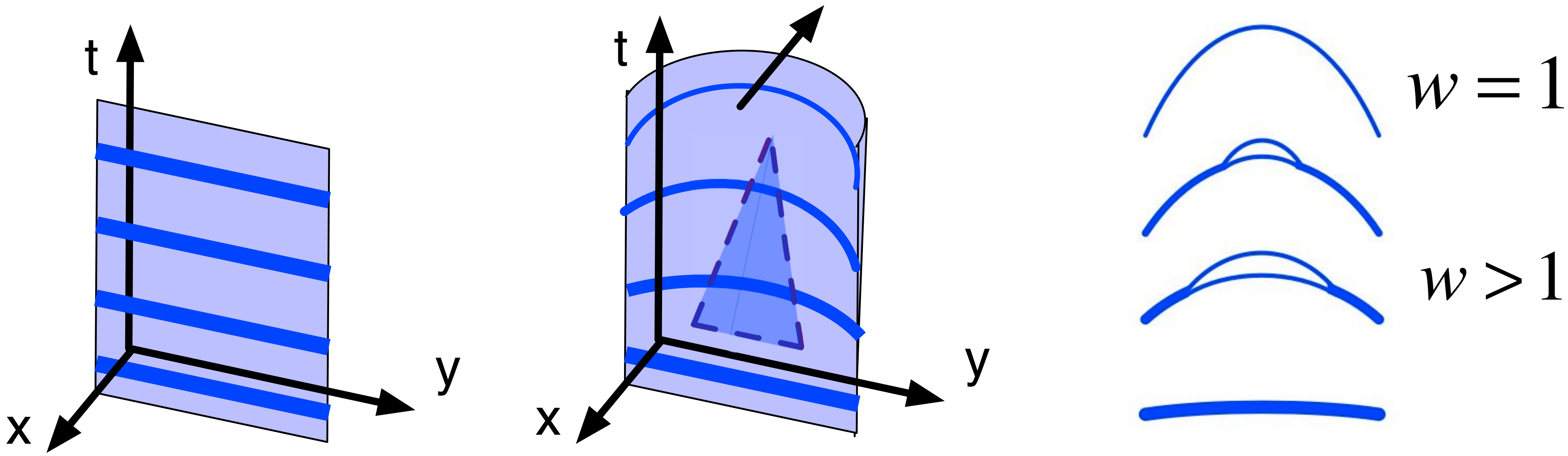
Branch weight



What prevents infinite branching?
Use branch weight $w = \sqrt{|\det g|} = \rho^4$ and require that w is preserved at each branching.

If M has total weight w when it is unbranched, then it cannot branch more than w times. ($w \geq 1$)

$$\hat{R}^{\mu\nu} = 0$$



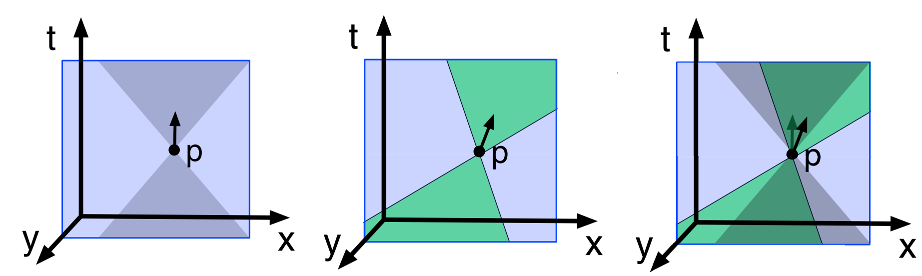
The Ricci flatness constraint on $g_{\mu\nu} = \rho^2 A^{\alpha}{}_{,\mu} A_{\alpha,\nu}$ allows an exchange between branching and volume.

Stretching the manifold reduces the weight.
Again, $w \geq 1$ and weight is preserved at branching.
Therefore increasing the volume decreases the number of possible branches.

Assumptions:

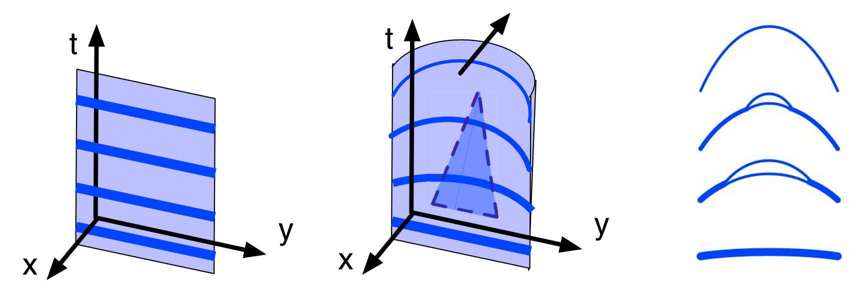
M is a branched 3+1-manifold.
M is embedded in a Minkowski 5+1-space.

A^{ν} is a vector field such that $\det(A_{\alpha,\mu}A^{\alpha}_{\ ,\nu}) = -1$.
ρ is a conformal weight.
Define the metric $g_{\mu\nu} = \rho^2 A^{\alpha}_{\ ,\mu} A_{\alpha,\nu}$.



The set g^{+} must intersect η^{+}.

The metric $g_{\mu\nu}$ is Ricci flat, $\hat{R}^{\mu\nu} = 0$.
Branch weight $w = \sqrt{|\det g|} = \rho^4$
is preserved at branching
and $w \geq 1$ everywhere.



Dynamics:

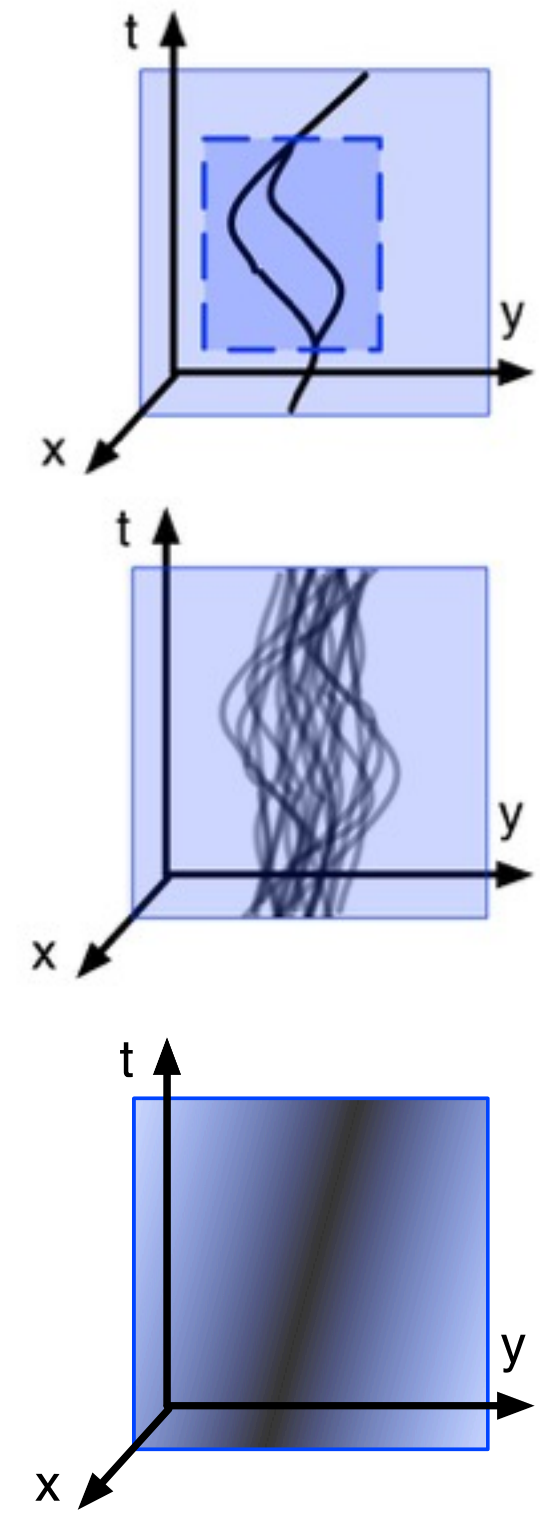
The spacetime manifold M is under-constrained.

The manifold maximizes entropy.

Calculation in principle

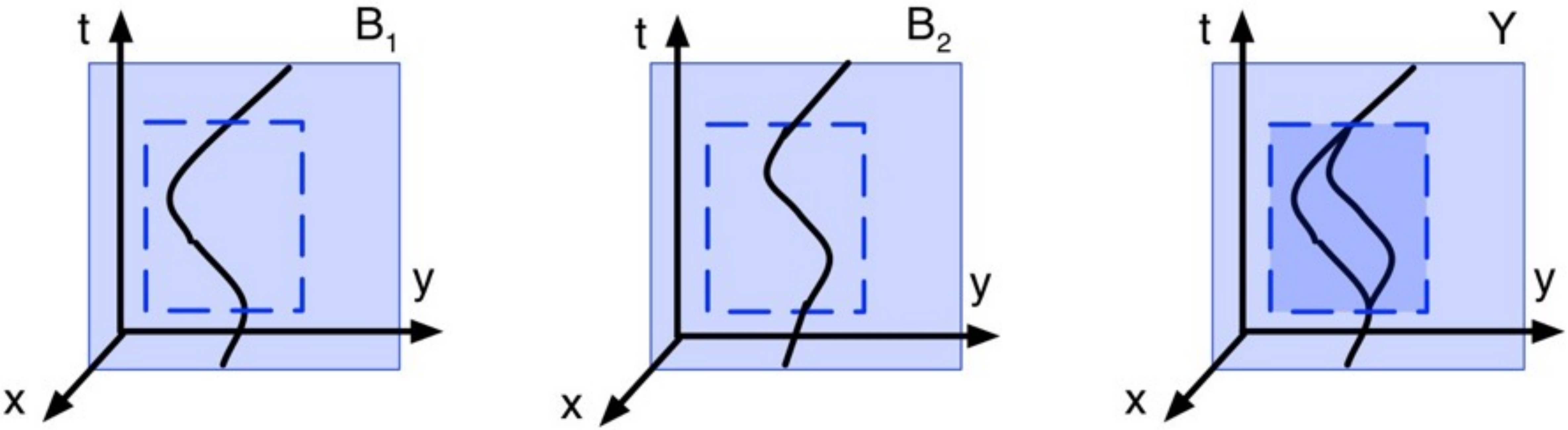
The spacetime manifold M maximizes entropy. Its branches continually split and recombine and cause knots on branches to recombine. Recombination of knots causes interference.

Knots have amplitudes. To calculate the effect of interference, we sum the knot amplitudes. The summed amplitude determines the probability.



Fermions

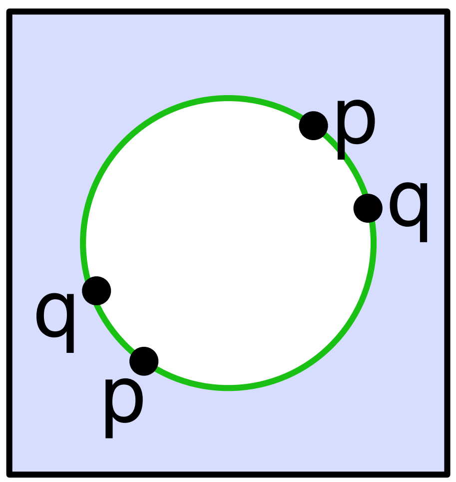
Knots

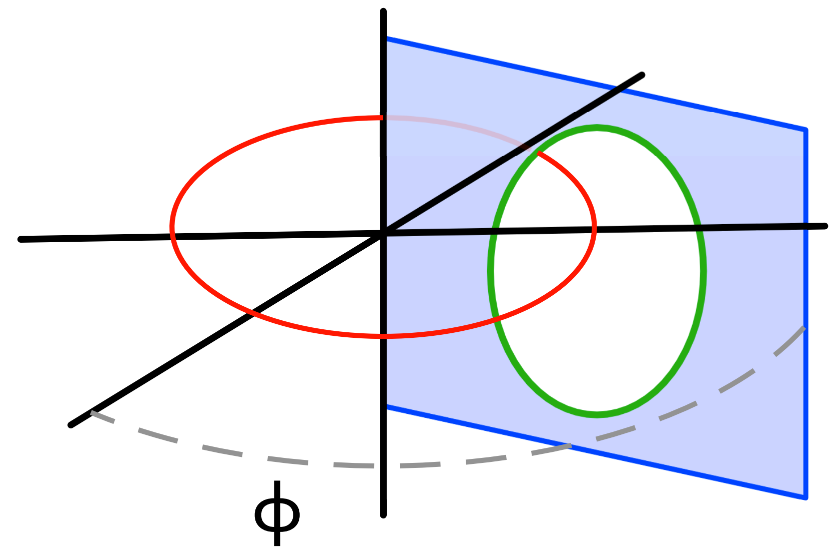


Elementary fermions are knots on the branches of M.

The topology of those knots is $\mathbb{R}^3 \#(S^1 \times P^2)$.

$\mathbb{R}^3 \#(S^1 \times P^2)$



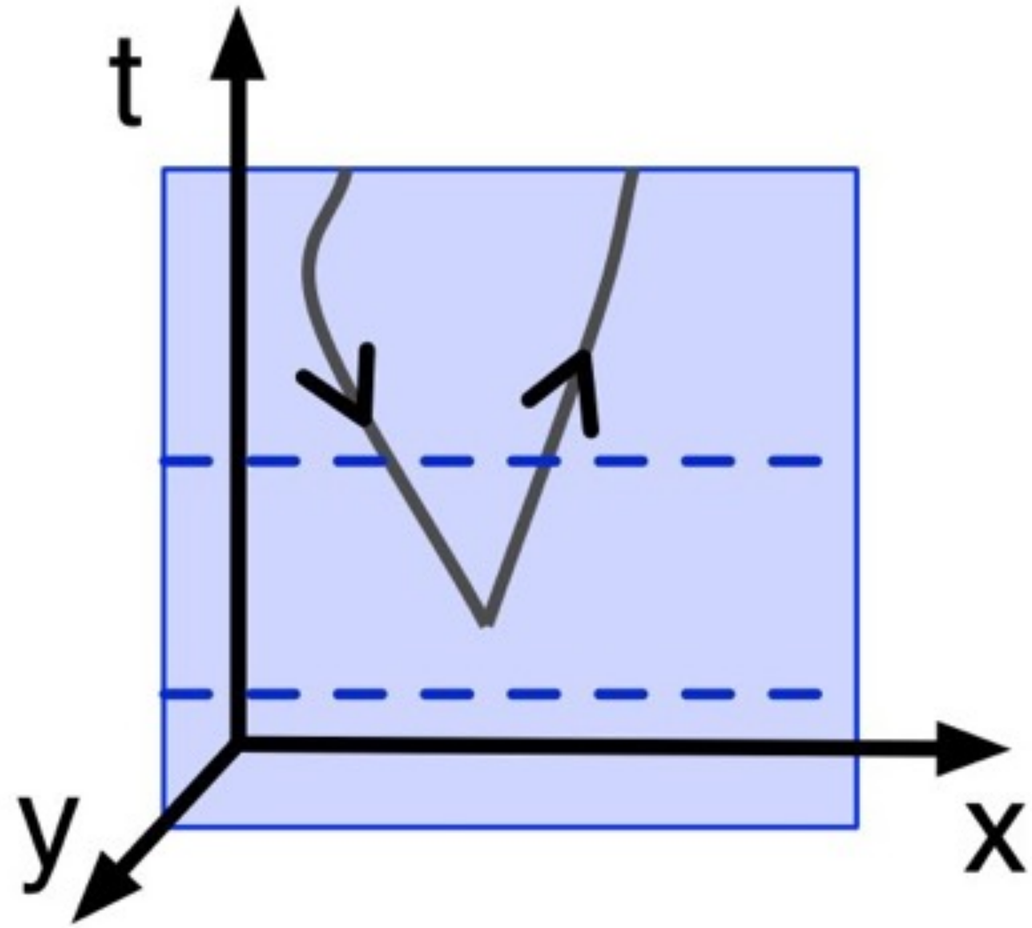


To make $\mathbb{R}^2 \# P^2$ cut a disk out of $\mathbb{R}^2$ and then glue together opposite points of the circumference.

To make $\mathbb{R}^3 \#(S^1 \times P^2)$ cut a torus out of $\mathbb{R}^3$ then glue together opposite points of the circumference.

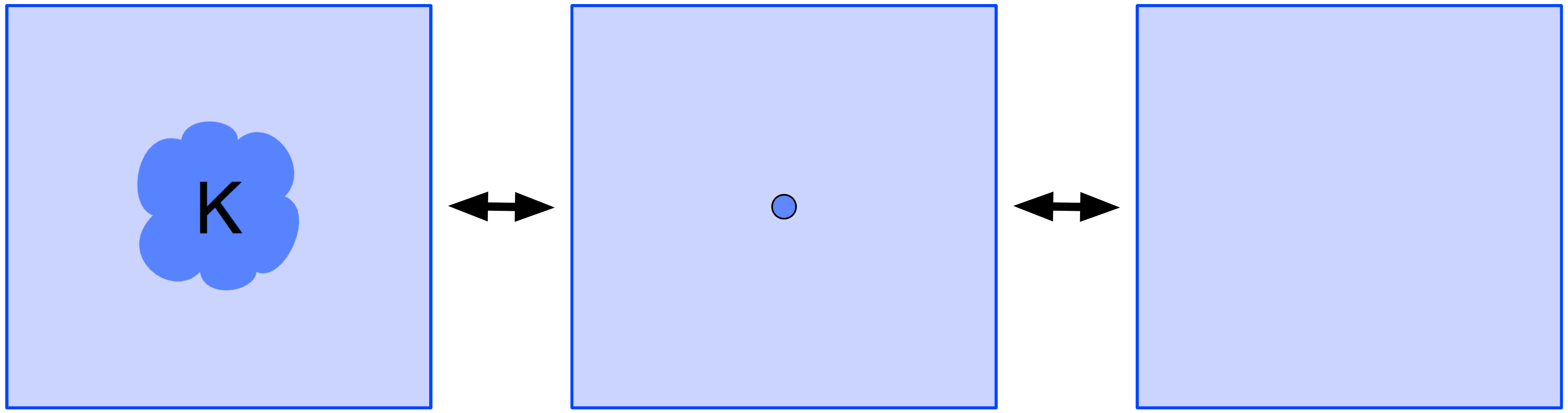
$\mathbb{R}^3 \#(S^1 \times P^2)$ is non-orientable, so use the spin group for the coordinate frame of fermions.

Topology change



If fermions are knots, then production of particle/anti-particle pairs implies a change of the topology of a spacelike slice of M.

Topology change

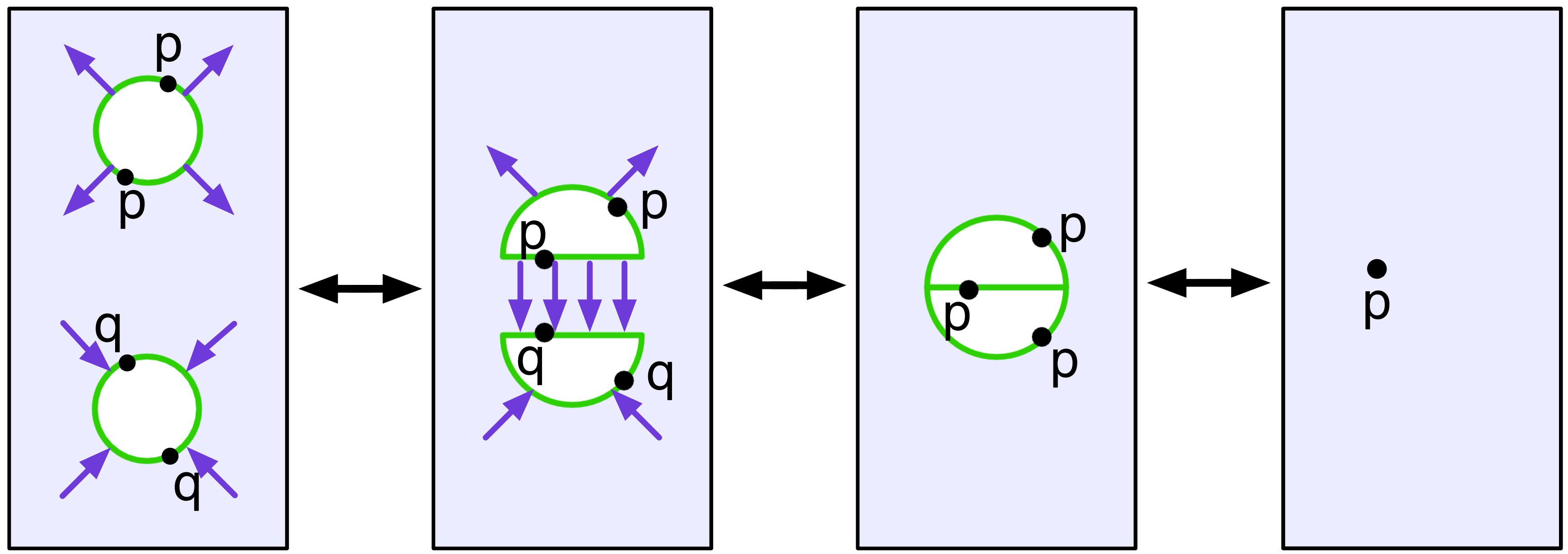


How can M change topology?

A knot on a manifold can be annihilated by shrinking it down to a point, but this requires the curvature to become infinite.

The metric $g_{\mu\nu} = \rho^2 A^{\alpha}{}_{,\mu} A_{\alpha,\nu}$ allows annihilation of pairs of $\mathbb{R}^3 \#(S^1 \times P^2)$ while preserving $\hat{R}^{\mu\nu} = 0$.

$\mathbb{R}^2 \# P^2$ Pair creation



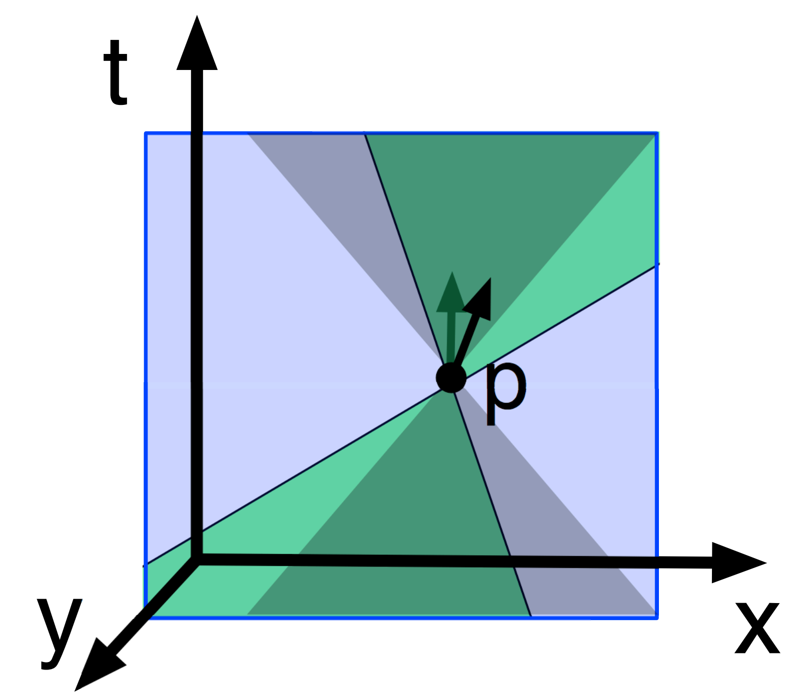
Derivatives of A^ν change the distance between a pair of $\mathbb{R}^2 \# P^2$ relative to $g_{\mu\nu} = \rho^2 A^\alpha{}_{,\mu} A_{\alpha,\nu}$.

When distance is zero, the point identification is the same as flat space, which is annihilation.

Pair creation is the same process in reverse.

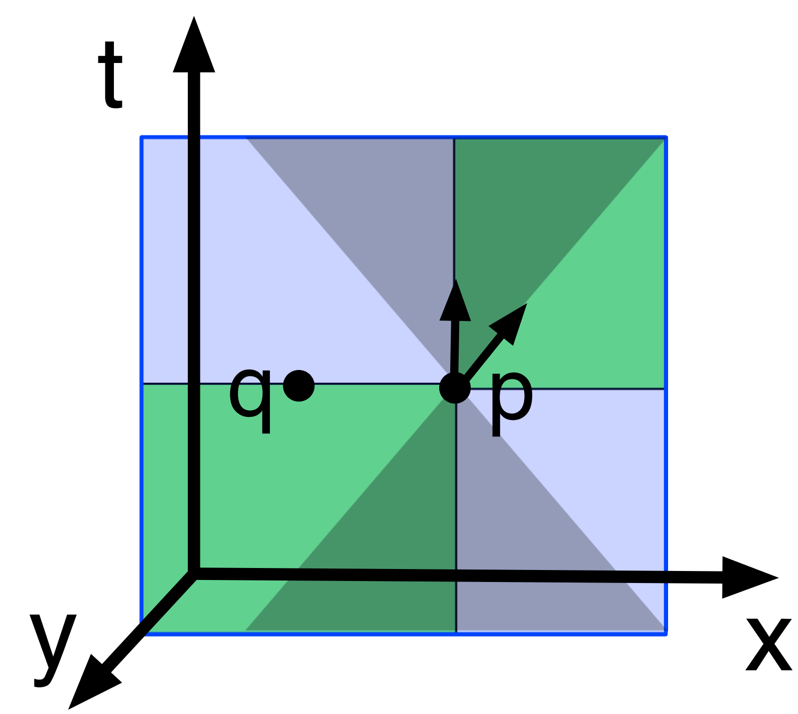
Topology change

The metric $g_{\mu\nu} = \rho^2 A^{\alpha}{}_{,\mu} A_{\alpha,\nu}$ uses the A^{ν} field. Changing the field changes the metric.

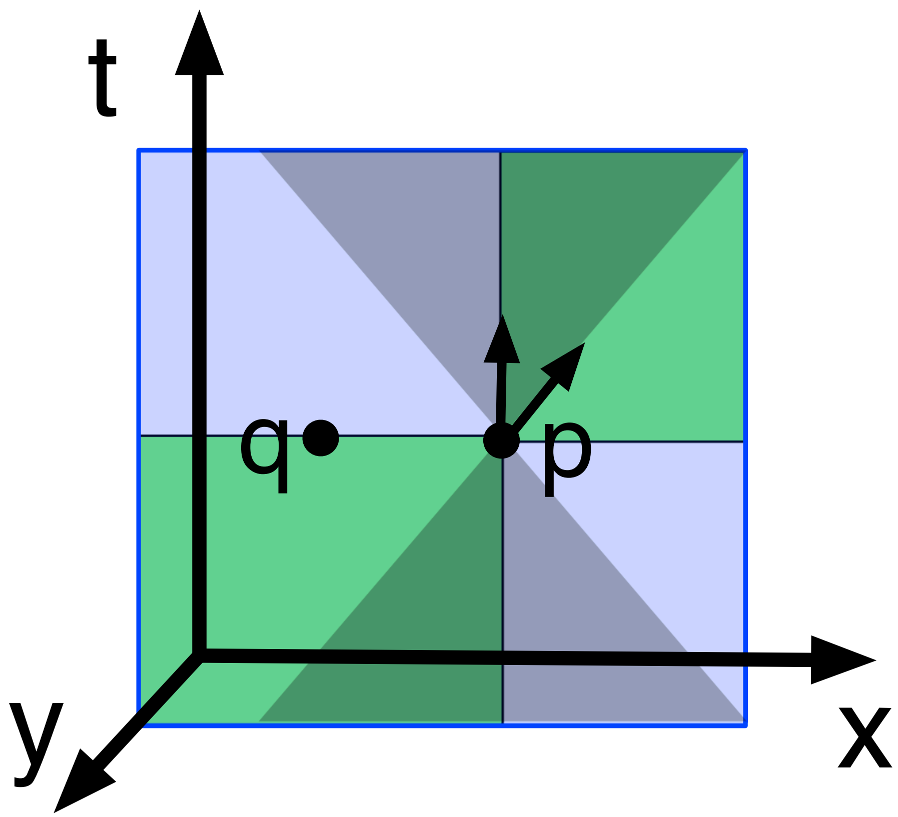


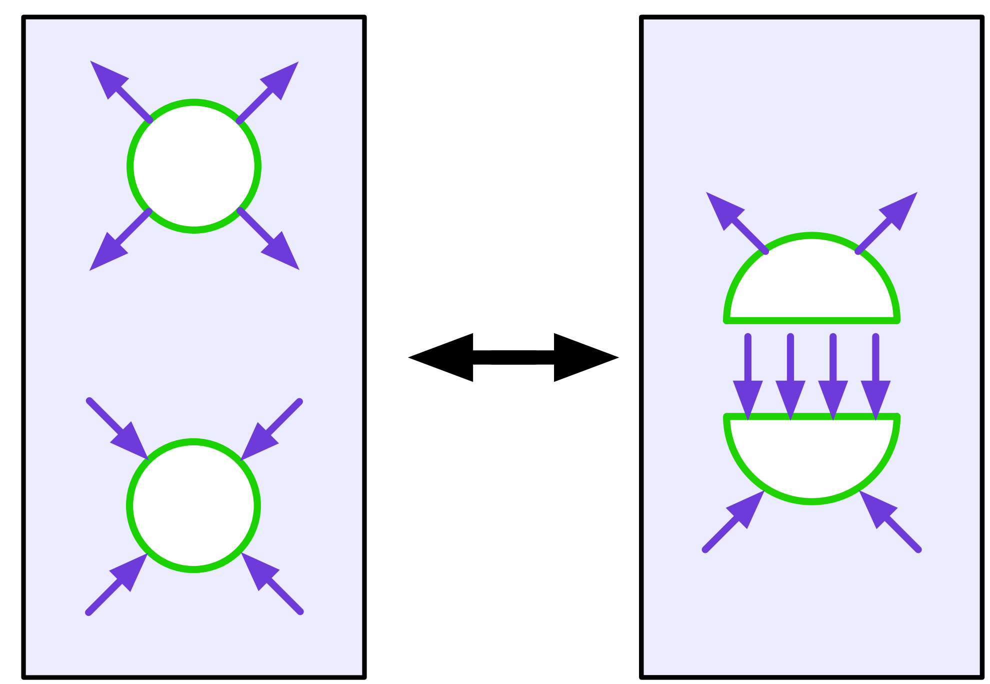
By changing $A^{0}{}_{,\nu}$ we can reduce distance between points to zero. For example,

$$g_{11} = \rho^2 (A^{0}{}_{,1} A_{0,1} + A^{1}{}_{,1} A_{1,1})$$



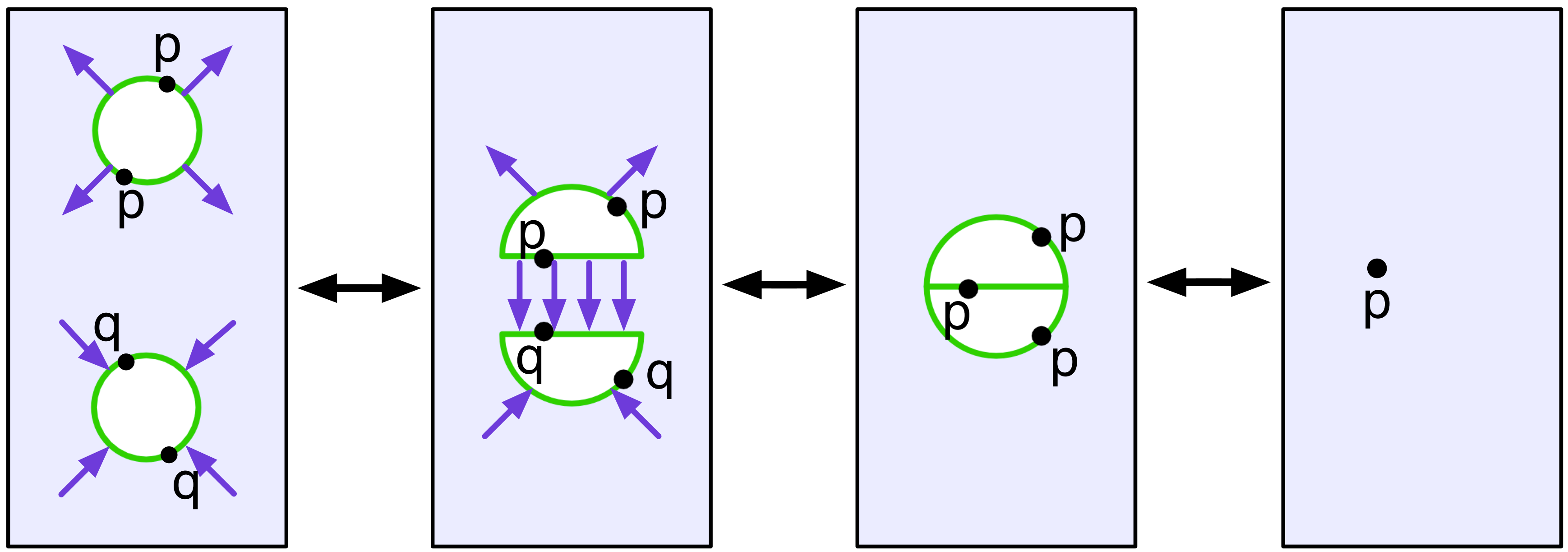
$\mathbb{R}^3 \#(S^1 \times P^2)$ Pair creation





We use $A^0_{,\nu}$ to reduce the distance $g_{\mu\nu} = \rho^2 A^\alpha{}_{,\mu} A_{\alpha,\nu}$ between a pair of $\mathbb{R}^2 \# P^2$ to zero.

$\mathbb{R}^3 \#(S^1 \times P^2)$ Pair creation

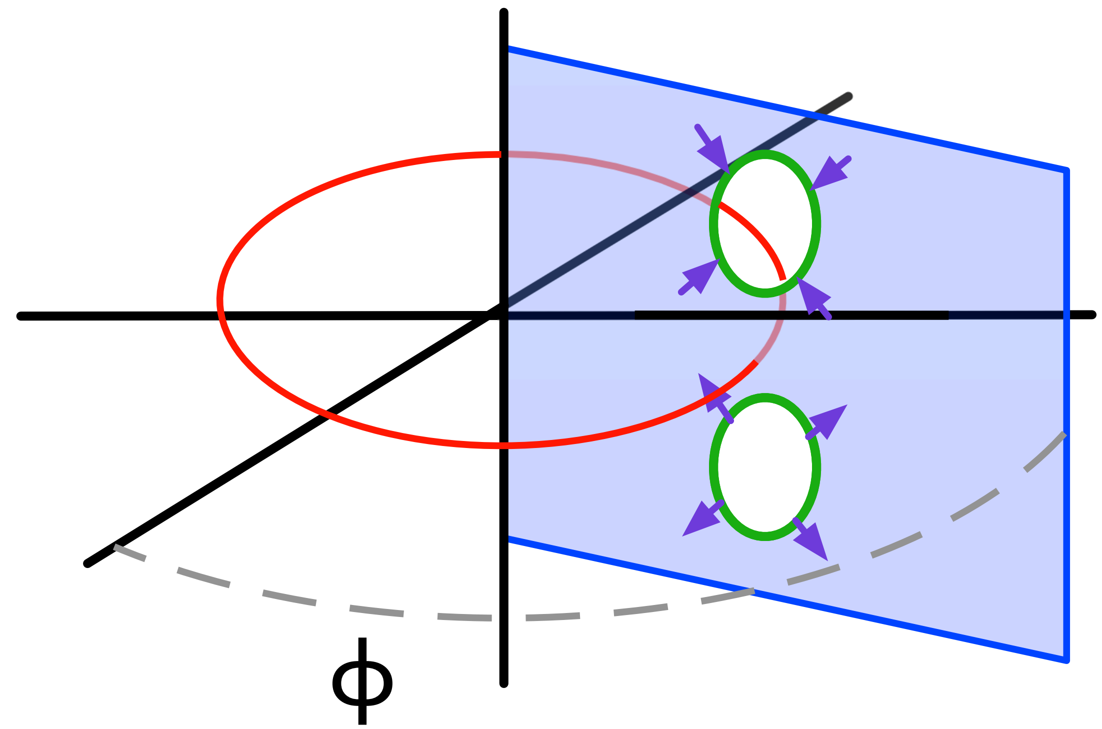


When the distance between them goes to zero, the point identification is the same as flat space.

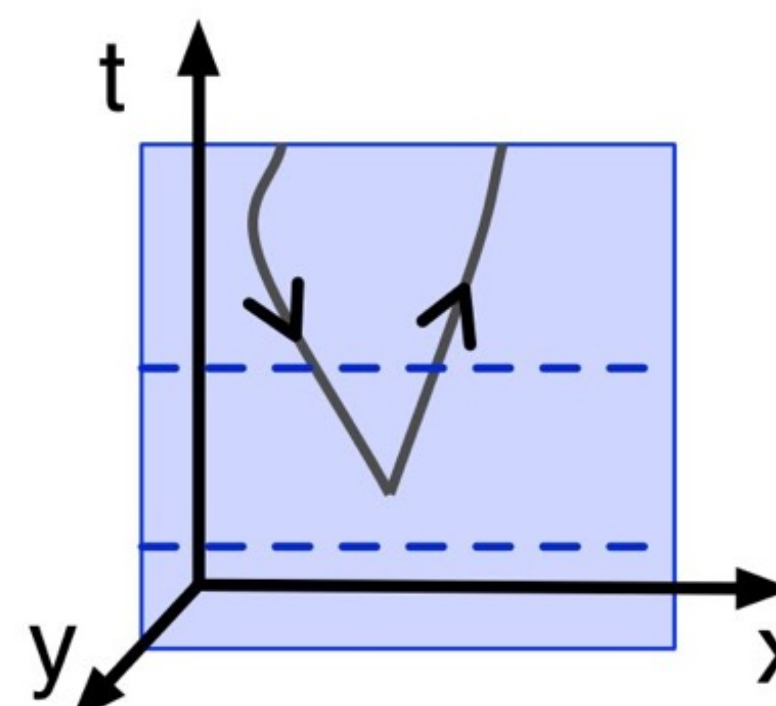
Then the pair of $\mathbb{R}^2 \# P^2$ have annihilated.

$\mathbb{R}^3\#(S^1 \times P^2)$ Pair creation

For $\mathbb{R}^3\#(S^1 \times P^2)$, fiber the annihilation over a circle. Pair creation is annihilation in reverse.

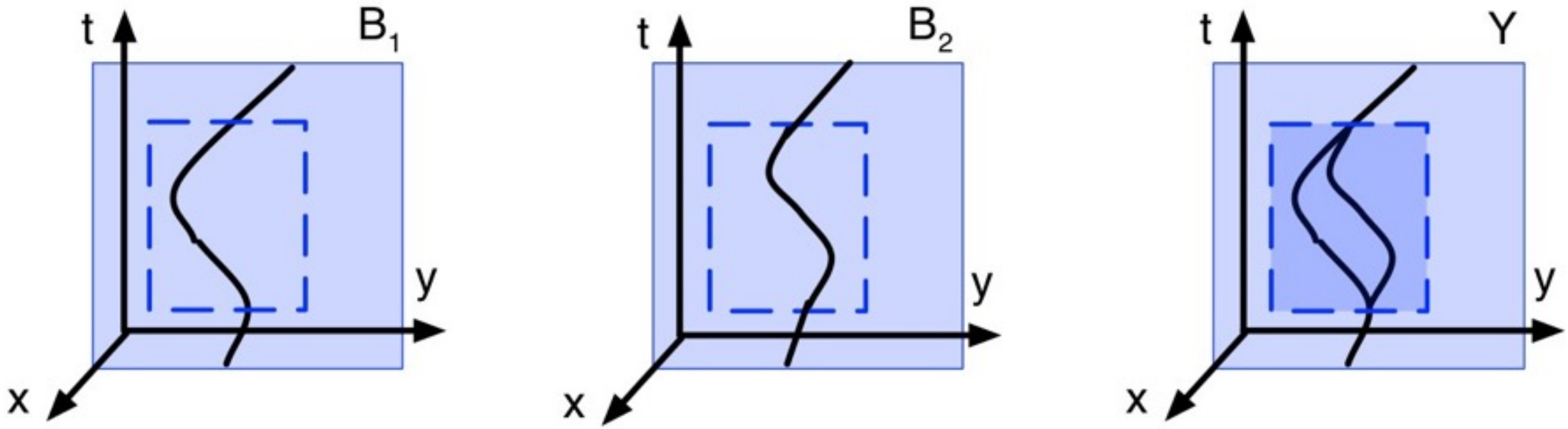


In this way we create and annihilate fermion pairs using the $A^0_{,\nu}$ field between them.



Quantum mechanics

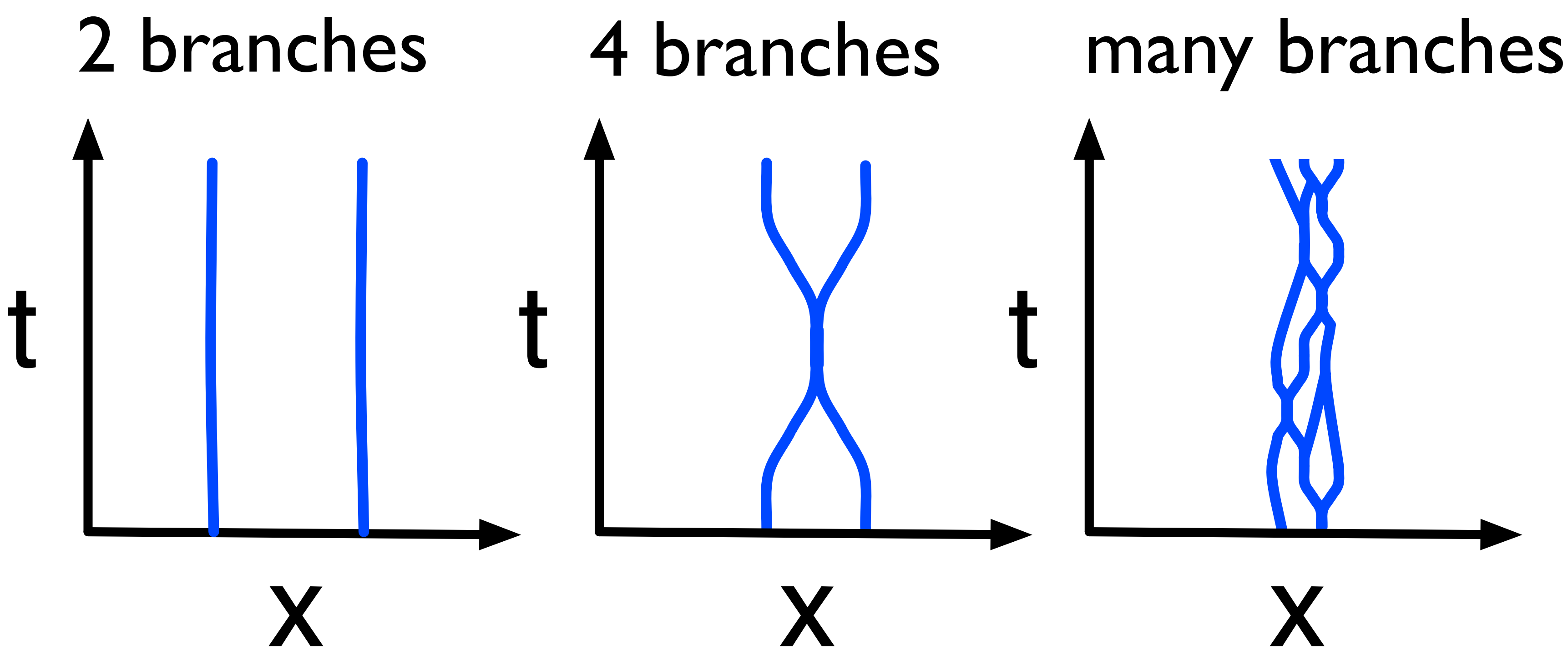
Quantum branches



To allow quantum mechanics, let M be a branched manifold. The branches separate and recombine.

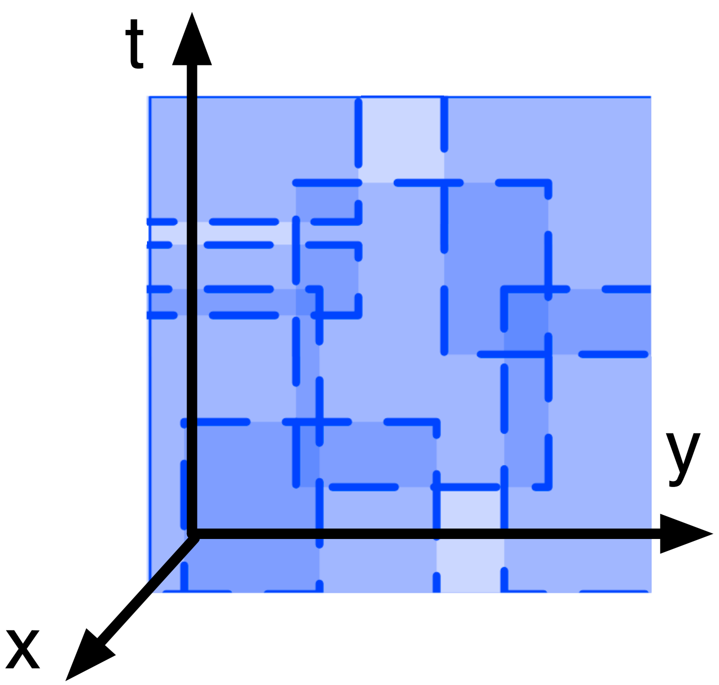
Knots on the branches also separate and recombine. Each Feynman diagram describes a class of branches.

Branch cohesion

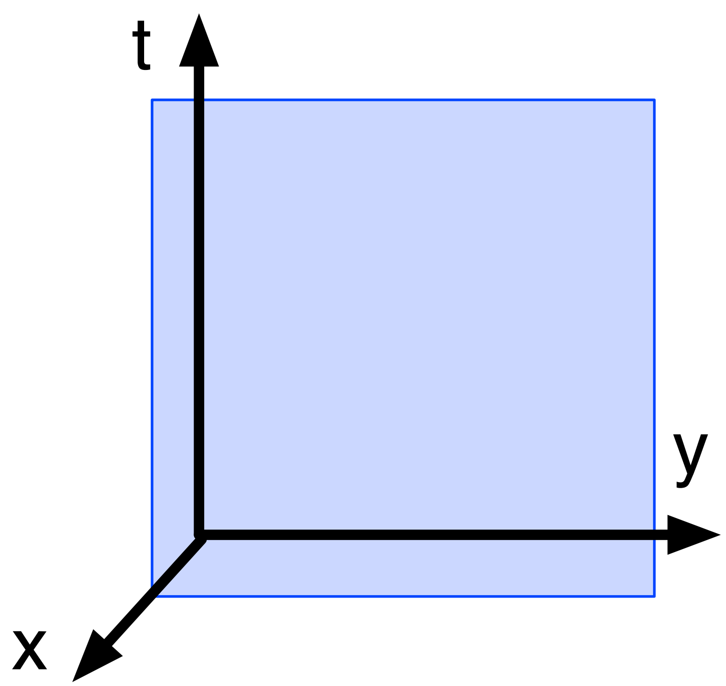


Recombination increases the number of branches.
Probability is proportional to the number of branches.
Therefore, maximizing entropy implies that branches stay close to each other and recombine often.
We will call this branch cohesion.

Φ_M the unbranched manifold



M, branched

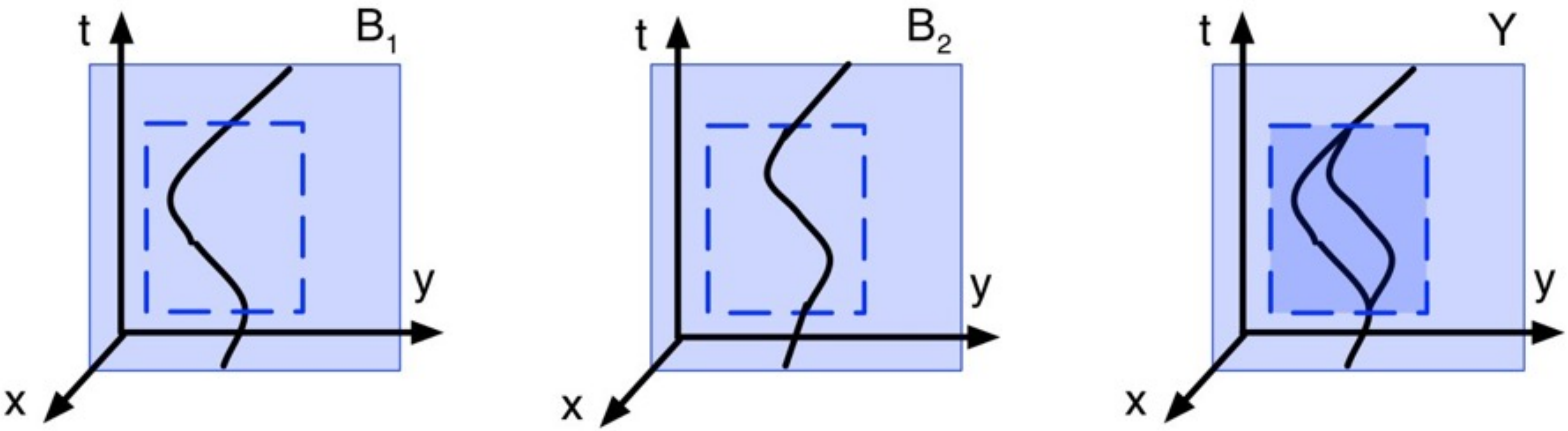


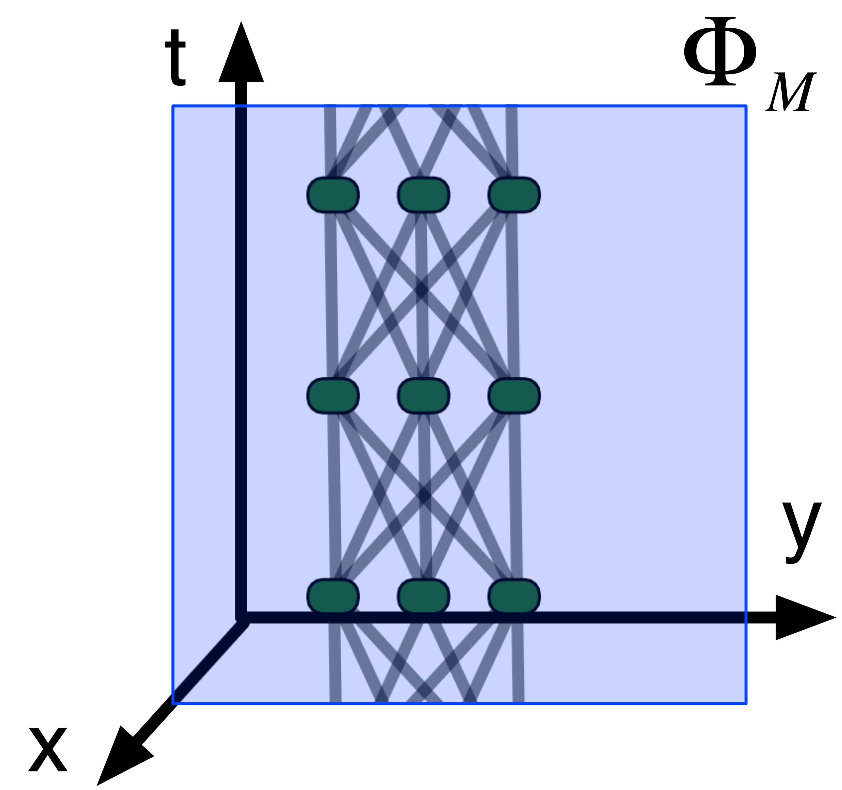
Φ_M, unbranched

M is a branched manifold, and it is complicated. Instead of describing all the branches and knots of M, we model M with an unbranched manifold Φ_M.

Define Φ_M as the unbranched manifold such that M is as close as possible to Φ_M.

Branching and paths

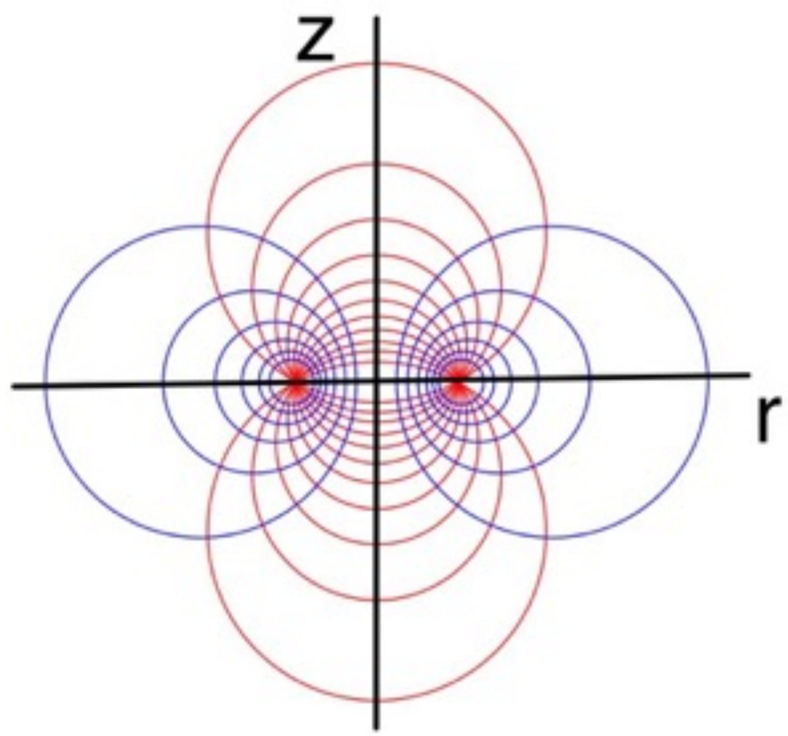


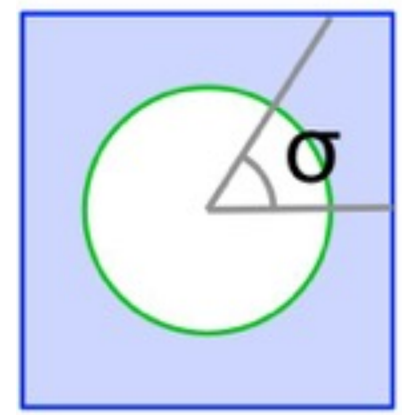


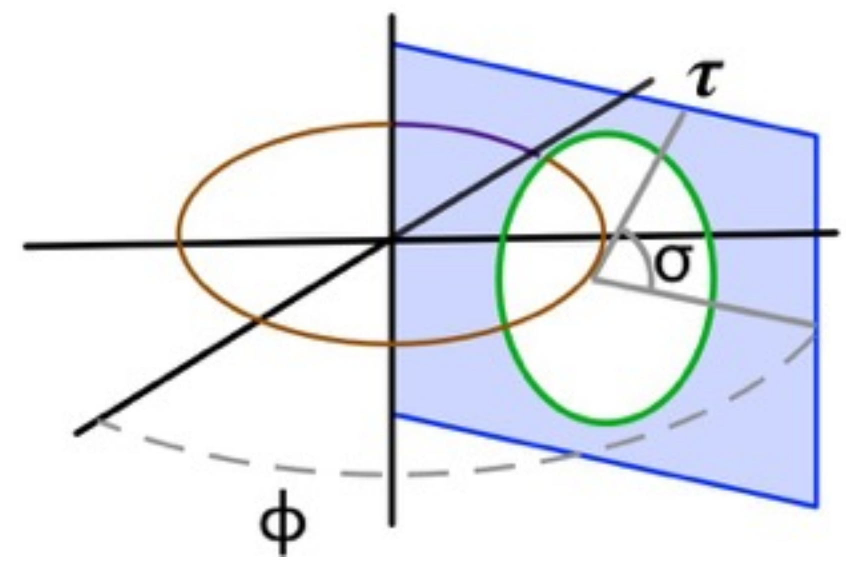
Knots on M take different paths on different branches.

We will describe knot geometry and interactions on M, then model the knots using Φ_M.

Toroidal coordinates

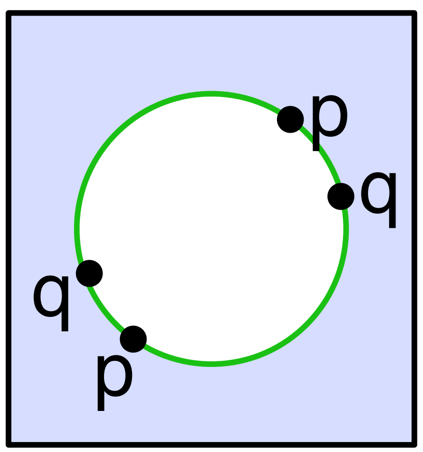


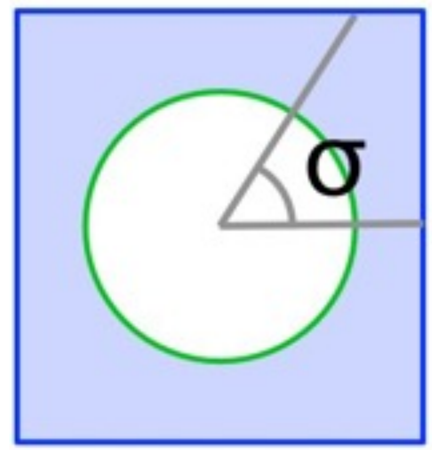


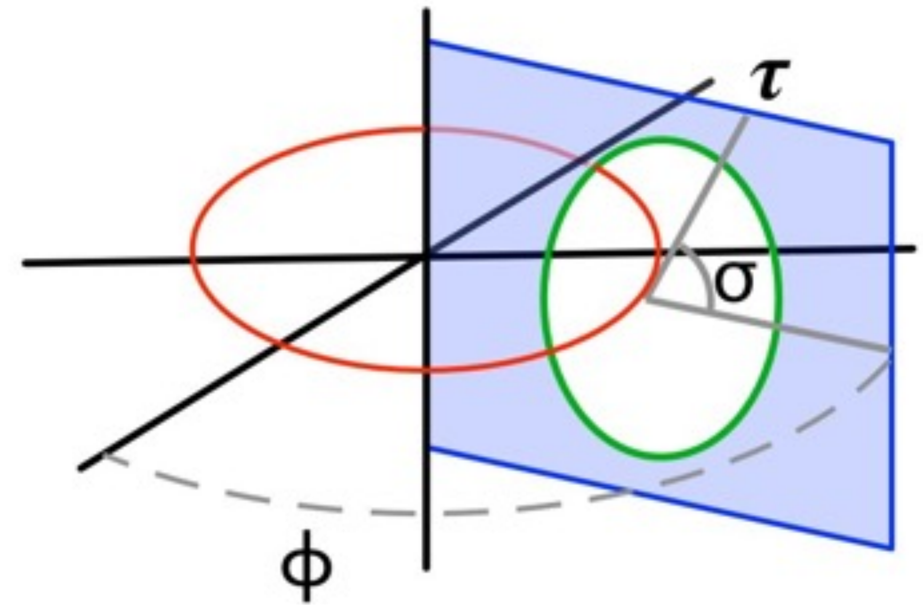


We will use toroidal coordinates to describe knot geometry as a map. In two dimensions, this is bipolar coordinates. Blue circles are a radial coordinate τ that goes to infinity as the blue circle gets small. Red circles are a coordinate σ that gives the angle at the points $(-1,0)$ and $(1,0)$.

Particle geometry







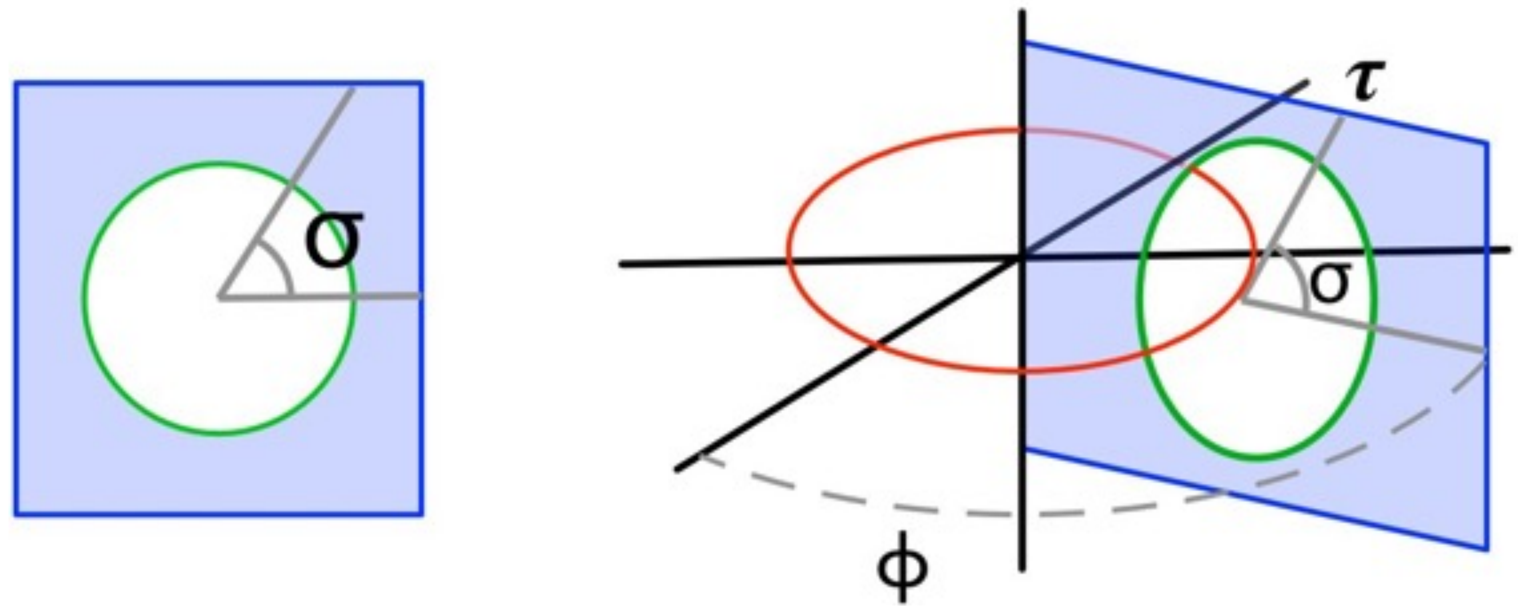
We describe knot geometry as a map in toroidal coordinates:

$$X:\mathbb{R}^3 - T = \{(\tau,\sigma,\phi);\tau \le 1\} \to \mathbb{R}^5 = \{(\tau,\sigma,\phi,x^4,x^5)\}$$

$$X(\tau,\sigma,\phi) = (\tau/(1-\tau),\sigma,\phi,\tau\sin(2\sigma),\tau\cos(2\sigma))$$

This is a first generation fermion.

Particle geometry



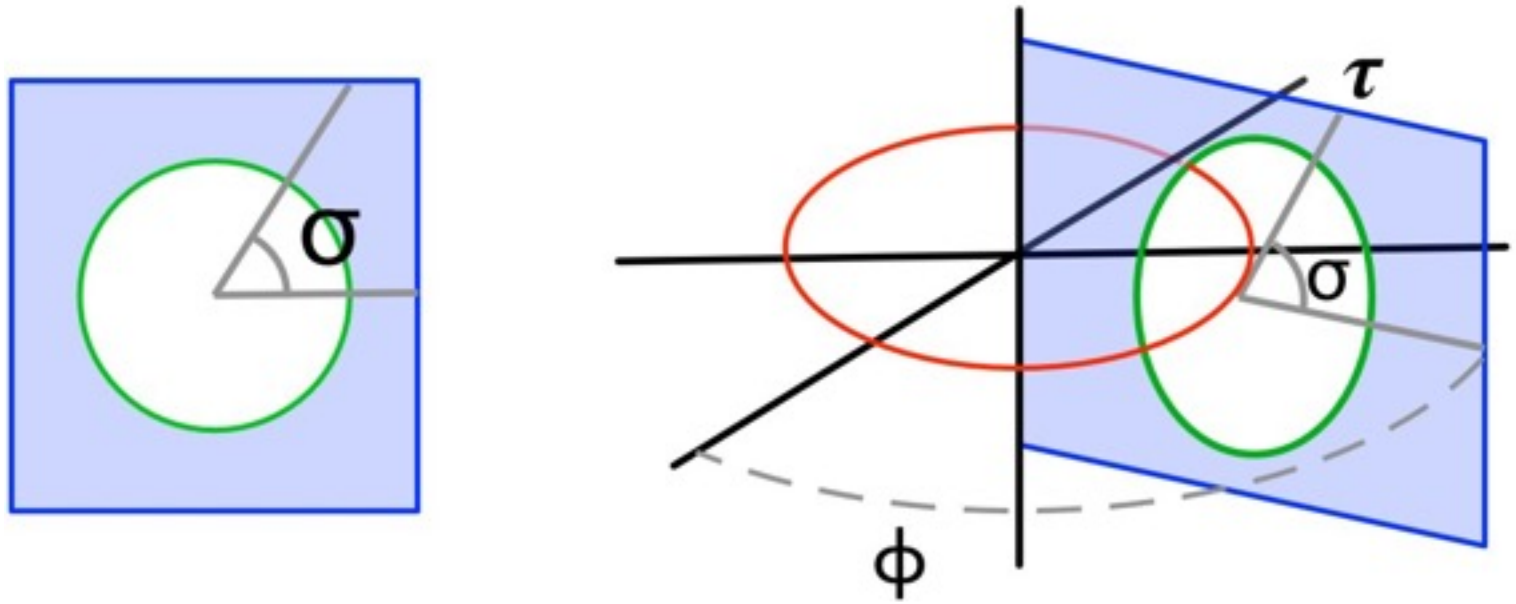
For a fermion of any generation:

$$X_n : \mathbb{R}^3 - T = \{(\tau,\sigma,\phi); \tau \le 1\} \to \mathbb{R}^5 = \{(\tau,\sigma,\phi,x^4,x^5)\}$$

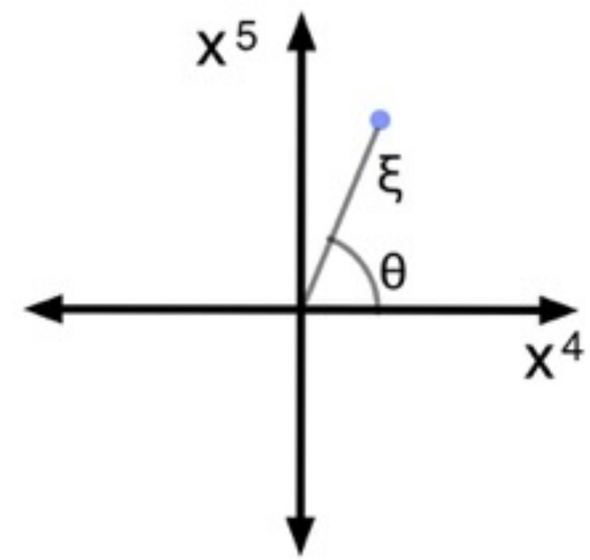
$$X_n(\tau,\sigma,\phi) = (\tau/(1-\tau),\sigma,\phi,\tau\sin(2\sigma+n\phi),\tau\cos(2\sigma+n\phi))$$

the generation is equal to n+1.

Particle amplitude



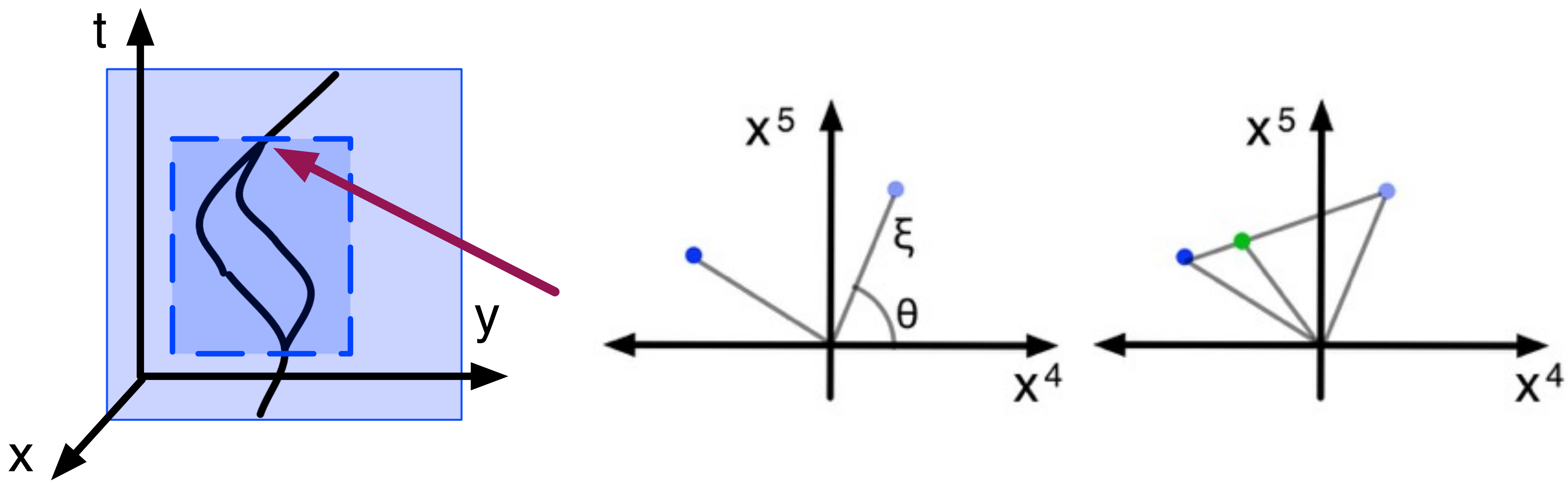
$R^3 \#(S^1 \times P^2)$ rotates in the co-dimension with phase angle θ_j and magnitude ξ_j.



Describe the geometry with a complex amplitude $a_j = \xi_j e^{i\theta_j}$

$$X(a_j;\tau,\sigma,\phi) = (\tau/(1-\tau),\sigma,\phi,\xi_j\tau\sin(2\sigma+\theta_j),\xi_j\tau\cos(2\sigma+\theta_j))$$

Particle interference

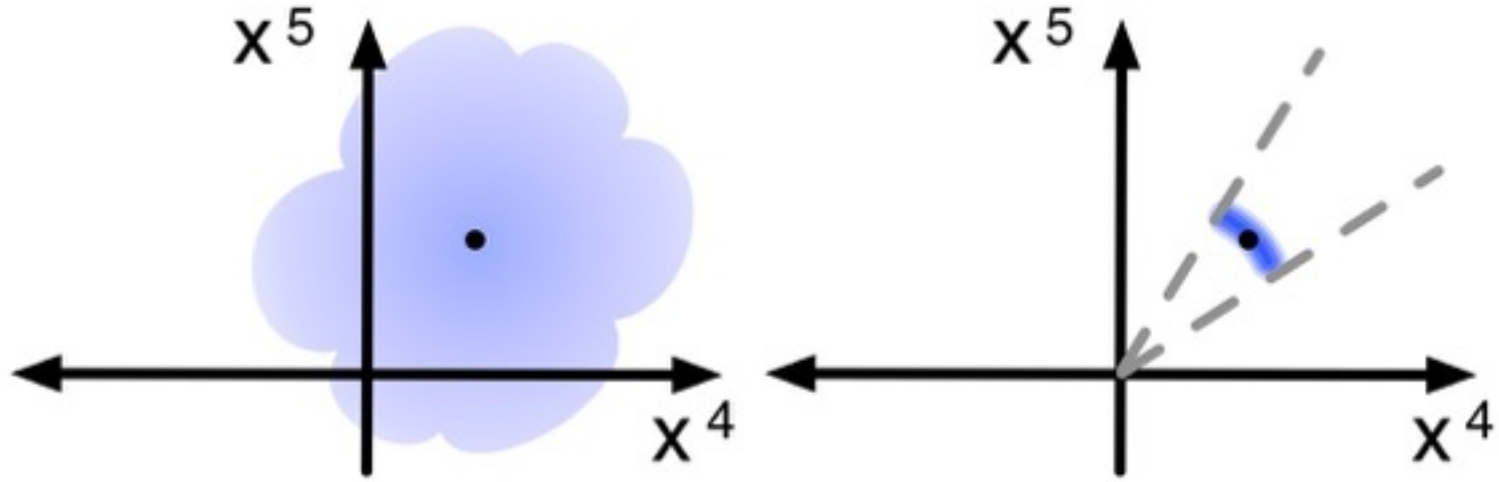


When branches recombine, the knots also recombine.
If original knots have a_j and w_j then

$$w_{sum} = \sum w_j \qquad a_{ave} = (\sum w_j)^{-1} \sum w_j a_j$$

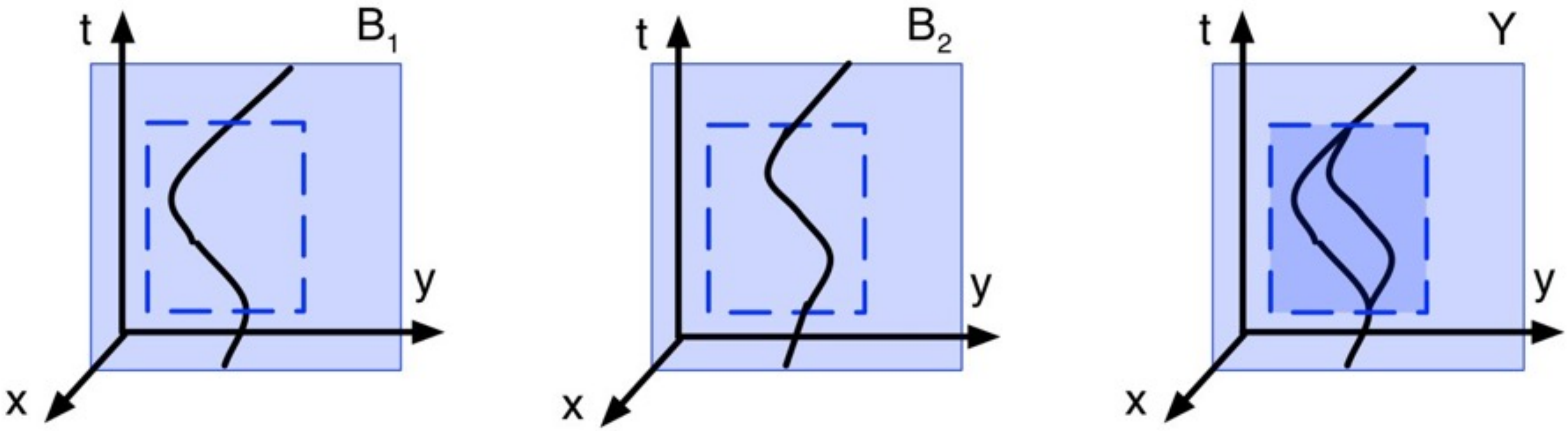
are preserved at recombination. If the branch splits again, the quantities are again preserved.

Particle interference

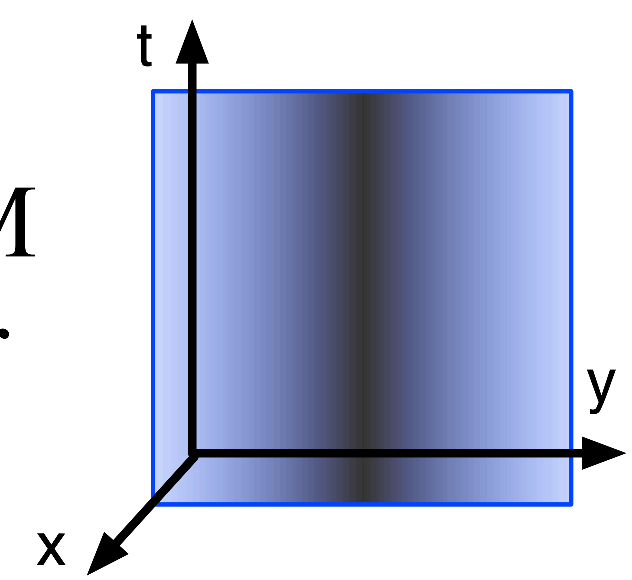


If many knots recombine, their complex amplitudes converge to an equilibrium distribution.
$w_{sum} = \sum w_j$ and $a_{ave} = (\sum w_j)^{-1} \sum w_j a_j$ are preserved at recombination.

Particle interference



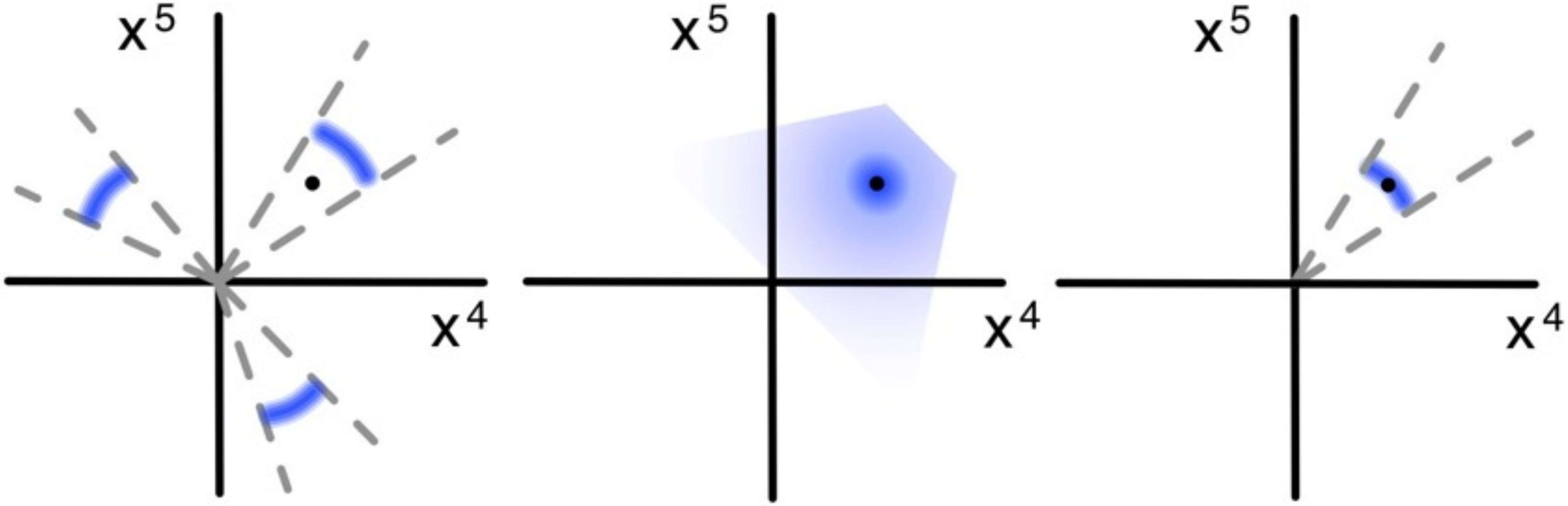
The branched manifold may have many branches and many knots.



On Φ_M we approximate the knots of M using $\psi = \sum w_j a_j$, where sum is over the branches of the manifold.

ψ is preserved at recombination because $\psi = w_{sum} a_{ave}$

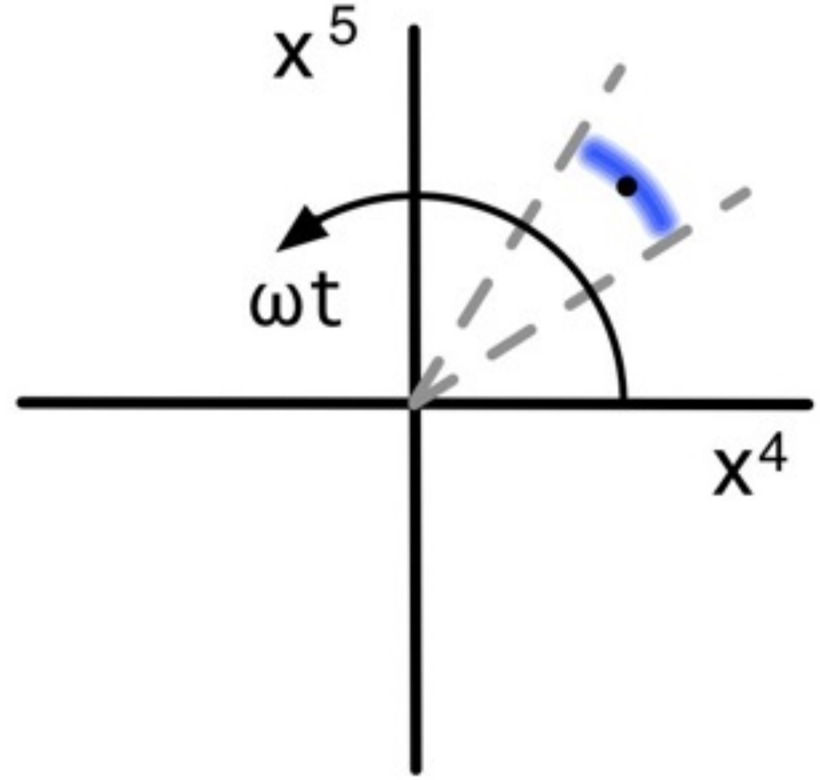
Particle interference



If multiple distributions recombine, the quantum amplitude of the union is the sum of the quantum amplitudes.

$$\psi = \sum \psi_m$$

Propagation in time

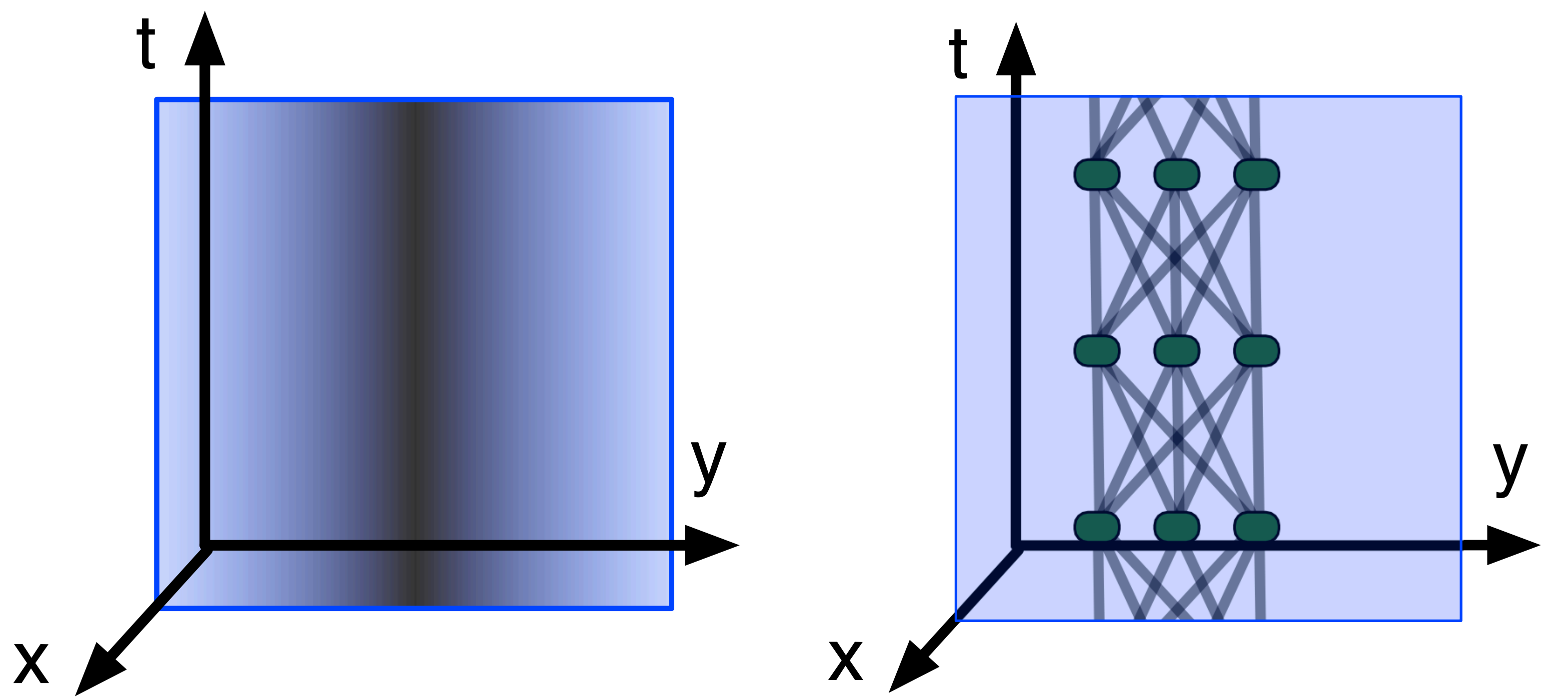


The quantum phase of the knots advances at a rate ω such that $E = \hbar\omega$ in the rest frame.

Then $\psi(t) = \sum w_j a_j(t) = \sum w_j \xi_j e^{i(\theta_j + \omega t)}$

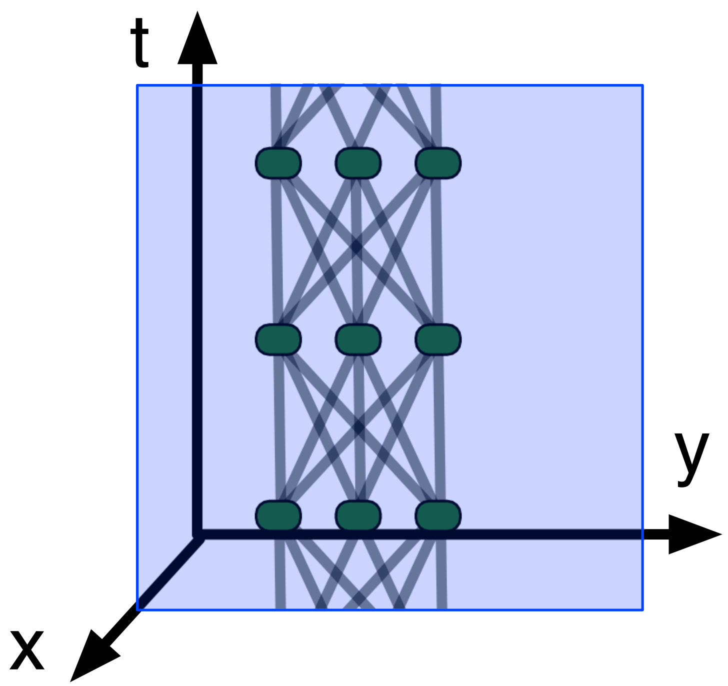
Quantum magnitude stays constant in time.

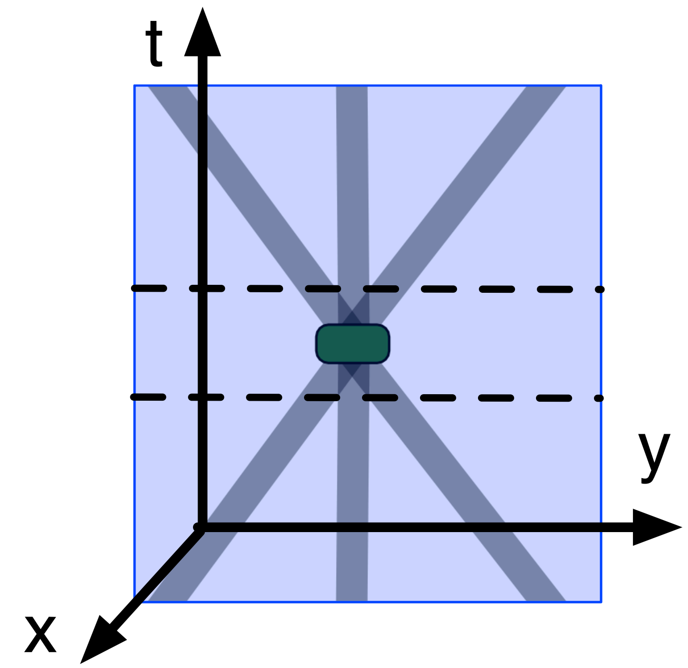
Quantum paths



To consider a simplified picture, we begin with a toy model where knots follow only a small number of possible paths and recombine only at a small number of locations at discrete times.

Quantum probability

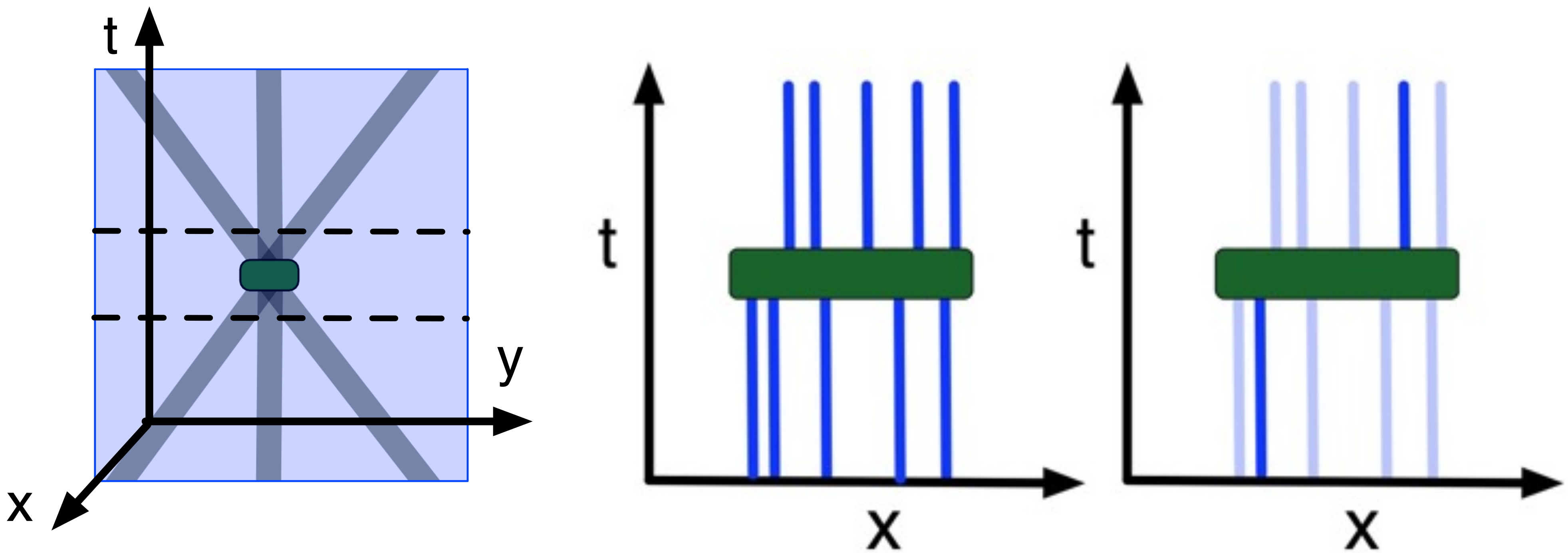




The probability of a particular event depends on the branches on which that event occurs.

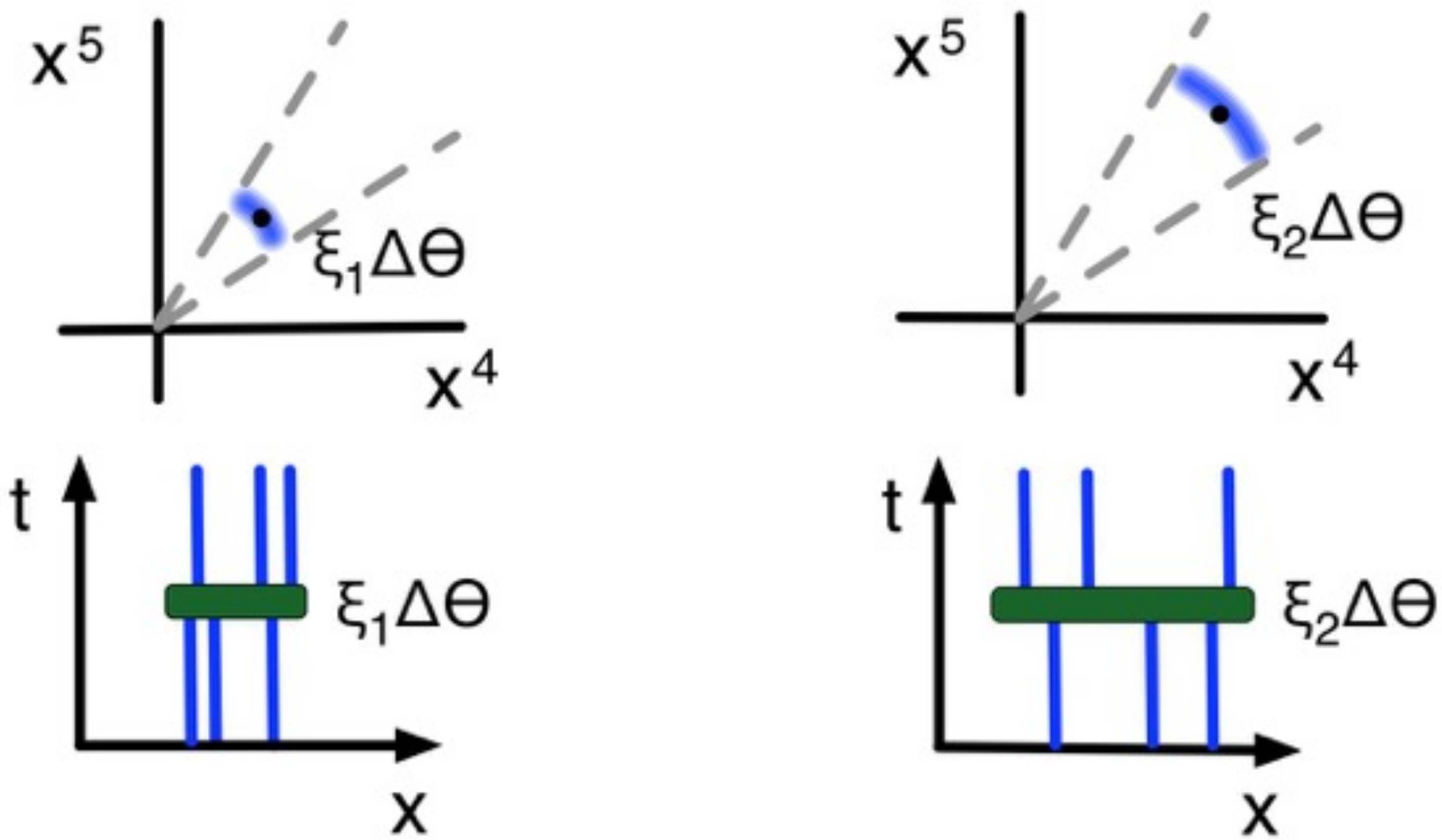
For each discrete knot path, there may be many branches that have a knot on that path.

Quantum probability



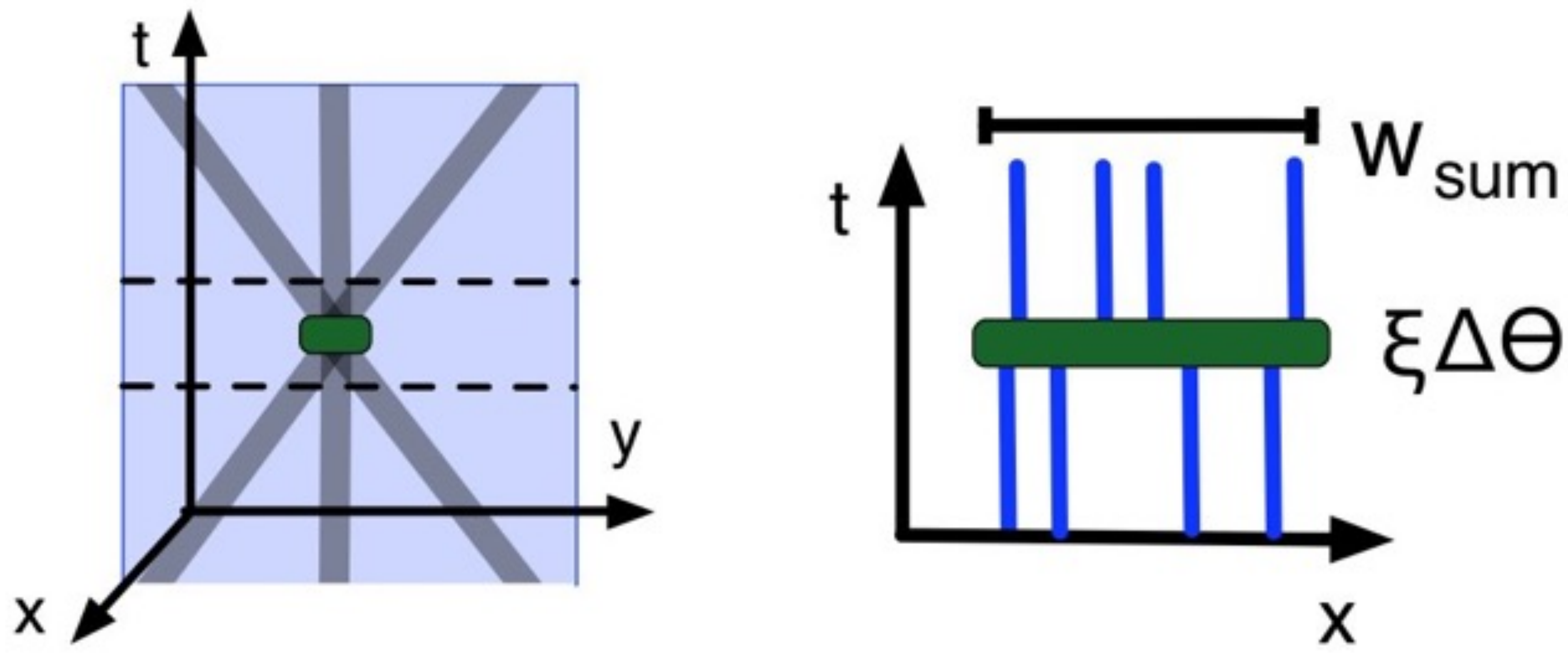
The number of ways for an event to occur is proportional to the number of branches coming into the event times the number going out. The number of branches is proportional to their total weight, $w_{sum} = \sum w_j$. Therefore $P \propto w_{sum}^2$.

Quantum probability



Probability is also proportional to the size of the state space of the branches coming into the event and going out. For an equilibrium distribution of knots, the size of the state space is proportional to the magnitude of the average amplitude. Therefore $P \propto |a_{ave}|^2$.

Quantum probability

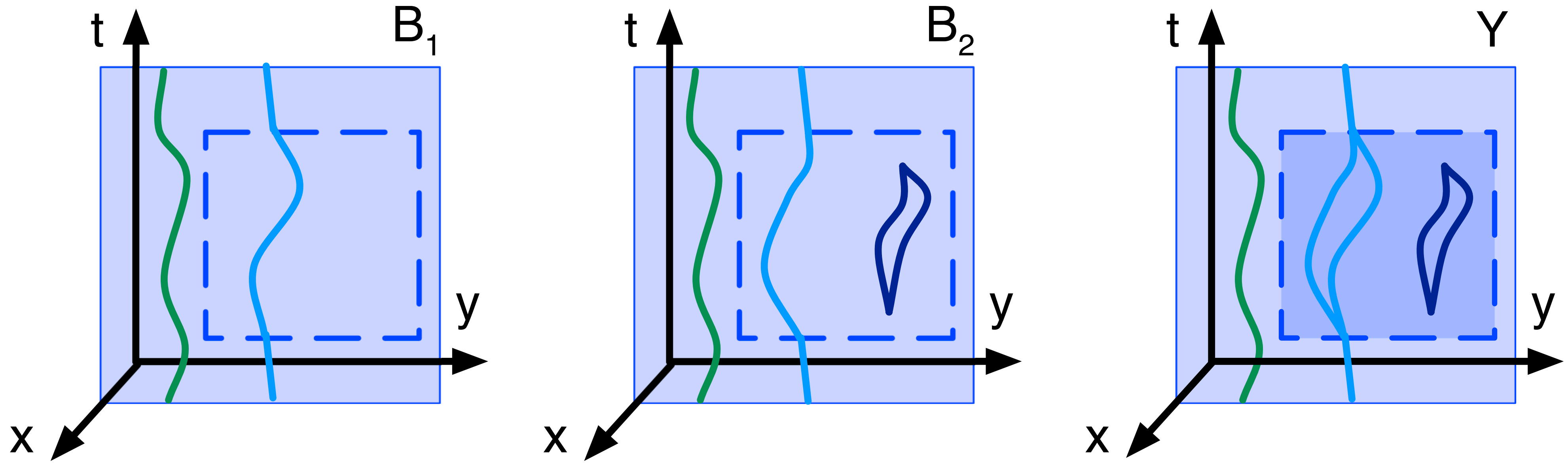


Combining the two, the probability of an event is
$P \propto |a_{ave}|^2$ and $P \propto w_{sum}^2$

Therefore $P \propto |w_{sum} a_{ave}|^2$

$$P \propto |\psi|^2$$

Virtual particles



A real particle has one knot on every branch of M.

A virtual particle has one knot on some of the branches of M.

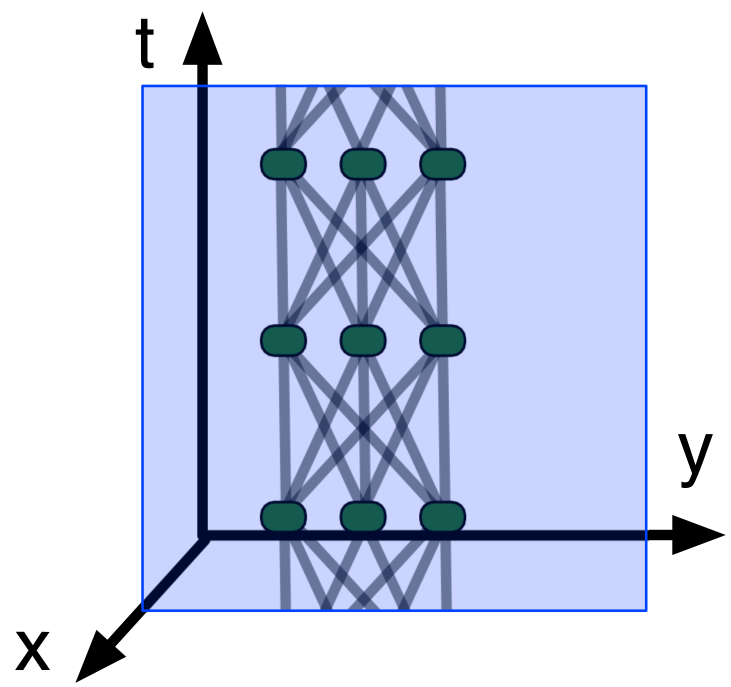
Particles (real and virtual) are created and annihilated subject to the metric $g_{\mu\nu} = \rho^2 A_{\alpha,\mu} A^{\alpha}_{,\nu}$ and $\hat{R}^{\mu\nu} = 0$.

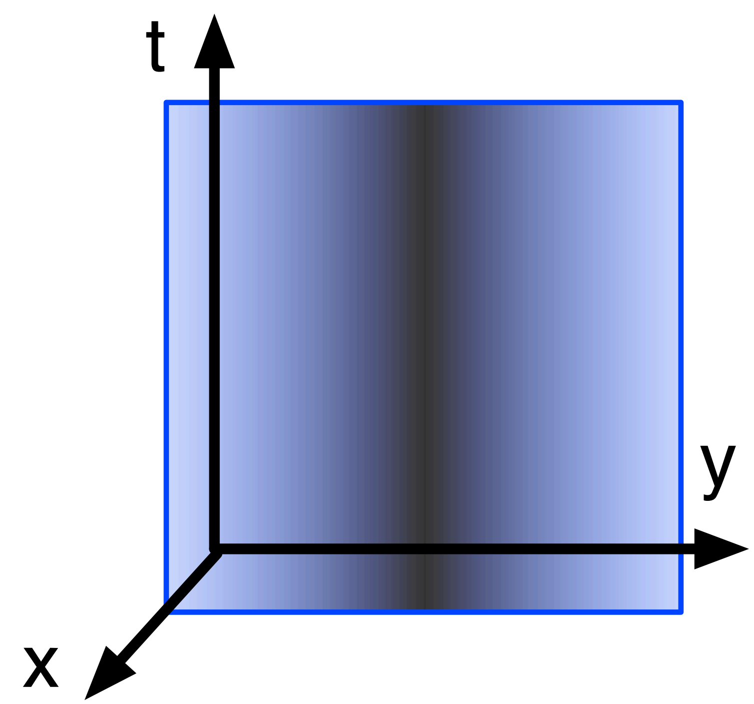
Path integration

We model the branched manifold M, including all particles, virtual and real.
Branches can be classified by Feynman diagrams.

For each knot type, we describe its distribution on M using a quantum amplitude ψ on Φ_M.

Path integration

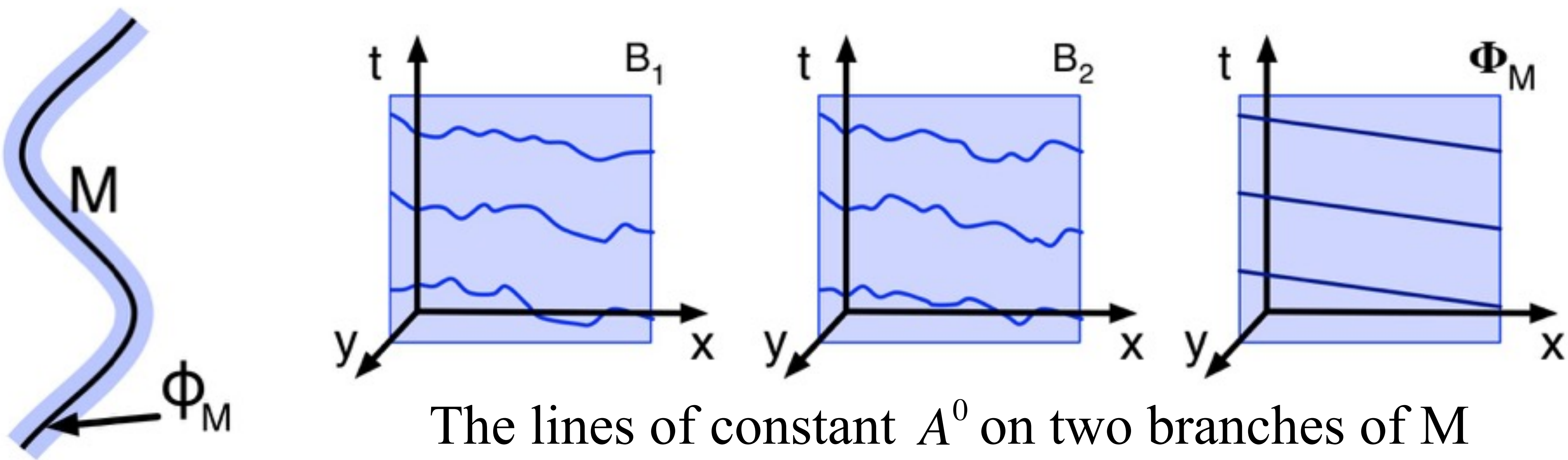




On M, the amplitude of a transition is the sum of amplitudes over all branches. This is a discrete sum. We calculate the discrete sum over branches of M by taking the continuous limit, which is the integral over all paths on Φ_M.

Lagrangian and entropy

Φ_M geometry, field, weight



The lines of constant A^0 on two branches of M and on Φ_M.

Φ_M is the unbranched manifold such that M is as close as possible to Φ_M, in both geometry and field A^ν.

The weight w on Φ_M is the sum of the branch weights on M.

Lagrangian definition

The manifold M is under-constrained.
The manifold maximizes entropy.

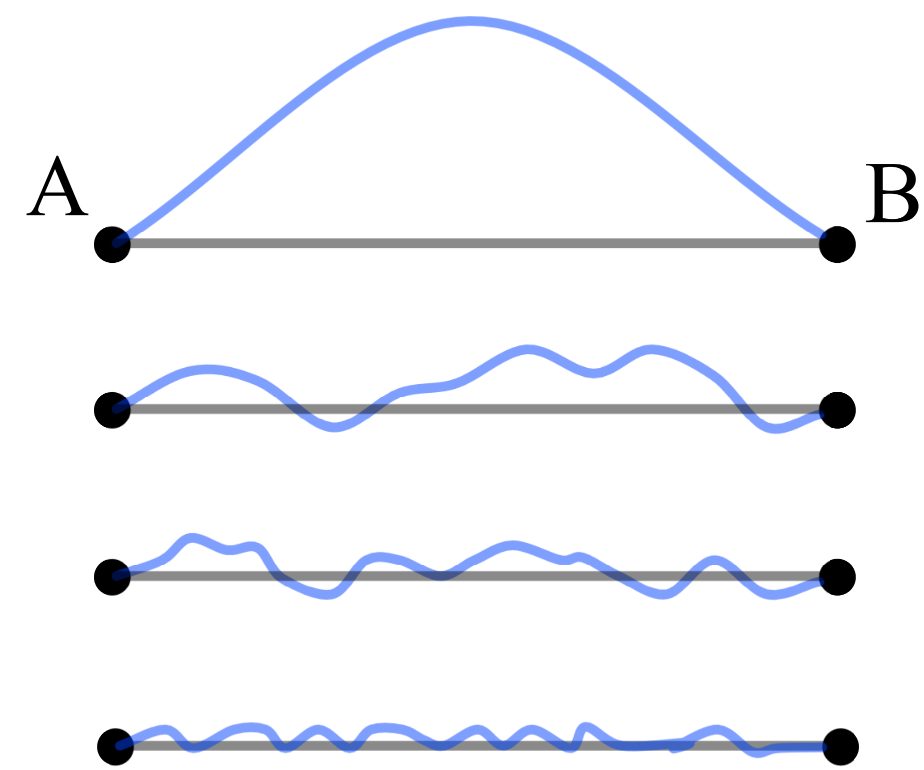
Define the Lagrangian L on Φ_M as the maximum entropy that M can achieve close to Φ_M.

Entropy on M is maximized when the Lagrangian on Φ_M is maximized.

Entropy of geometry

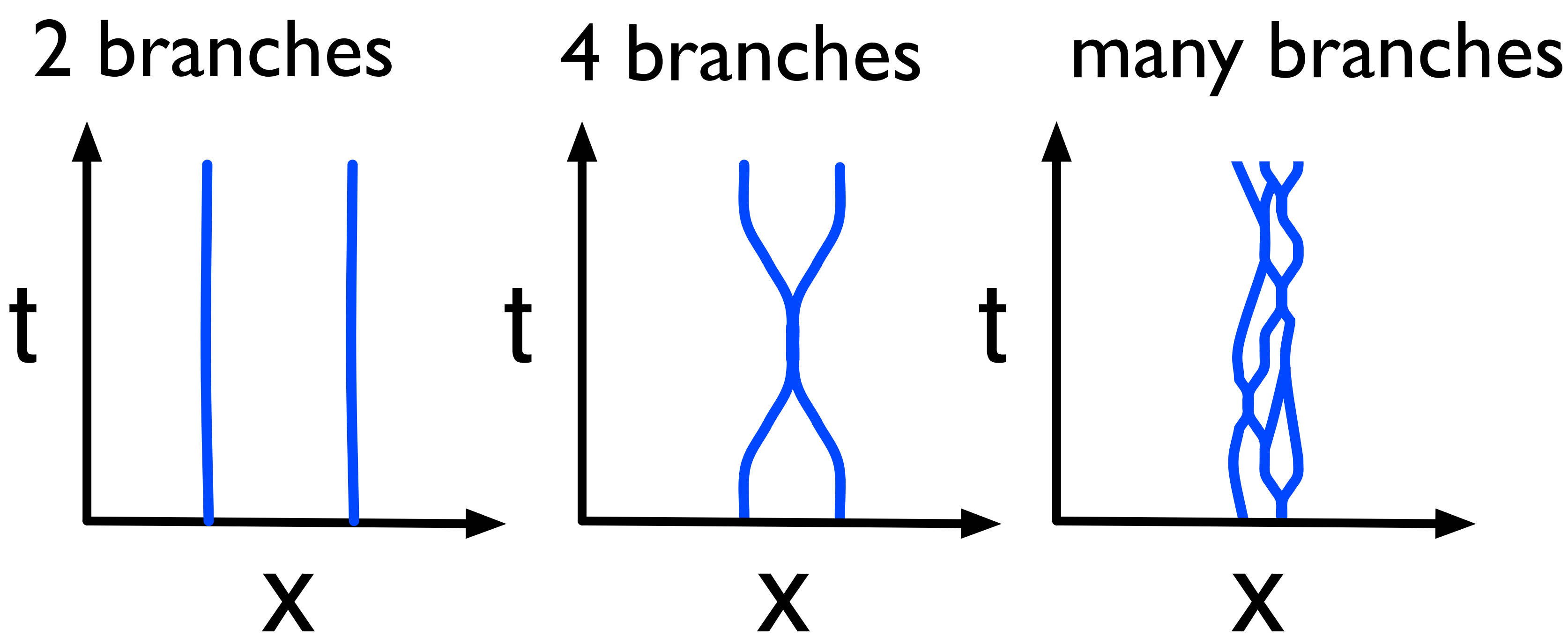
The manifold M has entropy in its geometry.

A 1-dimensional manifold with fixed length and fixed endpoints A and B maximizes entropy by being close to the straight line from A to B.



An n-dimensional manifold with fixed volume and fixed boundary maximizes entropy by being close to the minimal volume n-manifold.

Entropy of branching



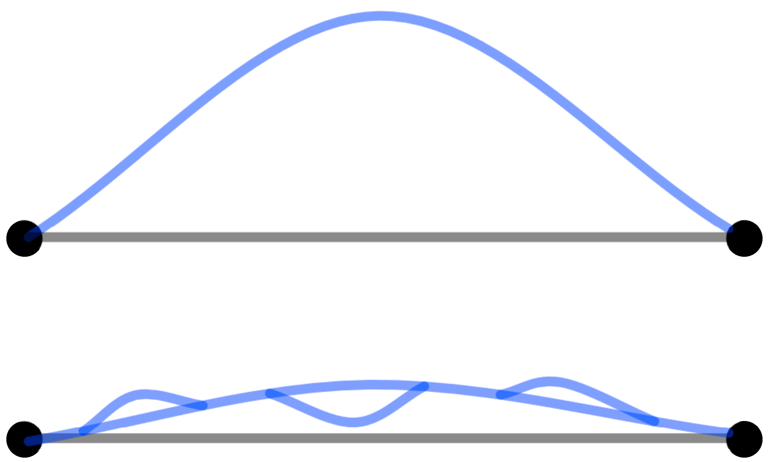
How does branching affect entropy?

The number of possibilities P is the number of branches.
The entropy is $S = \ln P$.

Entropy of branching

The manifold M has entropy in its branching.

A branched 1-dimensional manifold with fixed total weight and fixed endpoints maximizes its entropy by being close to the straight line between endpoints.

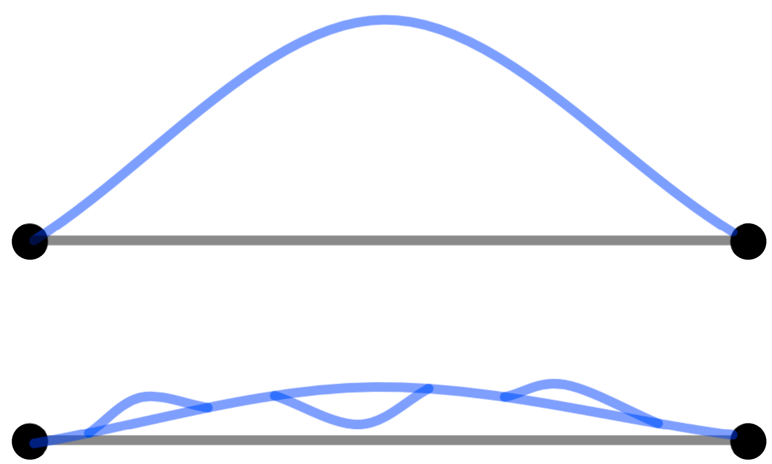


A branched n-dimensional manifold with fixed total weight and fixed boundary maximizes entropy by being close to the minimal volume n-manifold. The minimal manifold minimizes the scalar curvature R.

Entropy of branching

Entropy is linear in weight w.

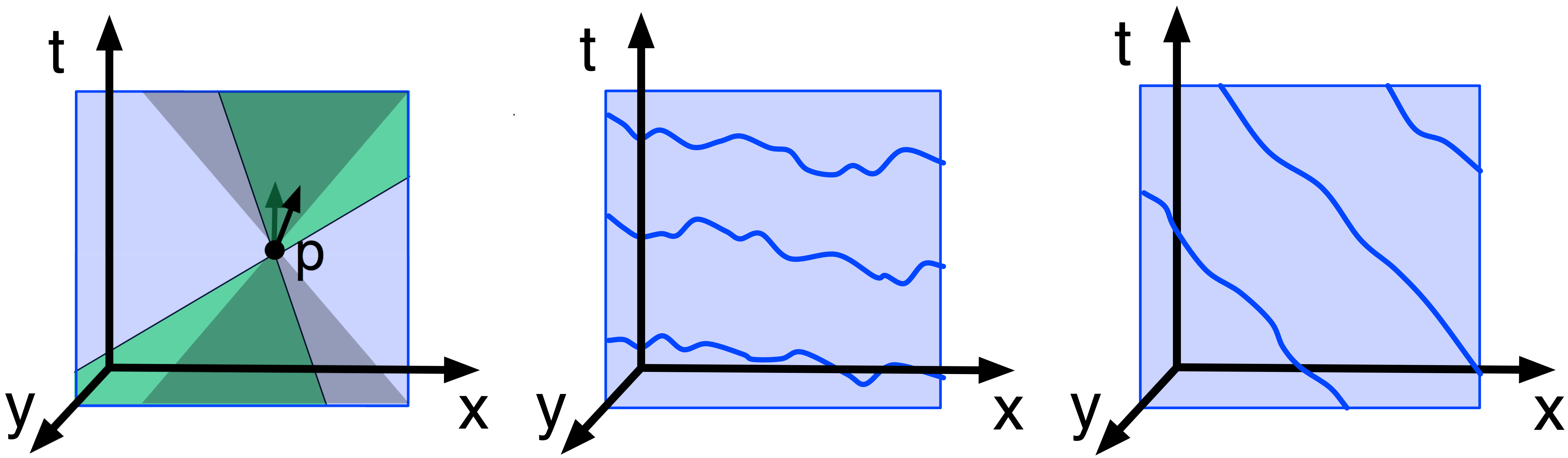
Let the maximal entropy be wS_0 for some constant S_0. Variation of the scalar curvature R gives entropy $S = wS_0 - wR$.



We defined the Lagrangian as the entropy. Therefore $L = wS_0 - wR$
Considering only the scalar curvature,

$$L = -wR$$

Entropy of A^ν

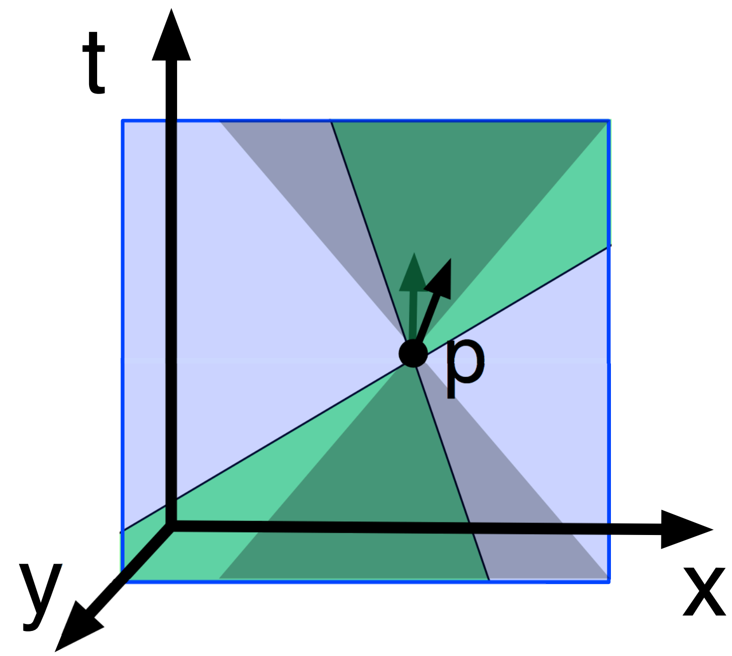


There is entropy in the A^ν field. The field is under-constrained, allowing for field fluctuations (virtual photons). However, constraint of g^+ constrains the magnitude of the field derivatives.

If there is an electric field, the entropy of the field fluctuations is reduced.

Entropy of A^{ν}

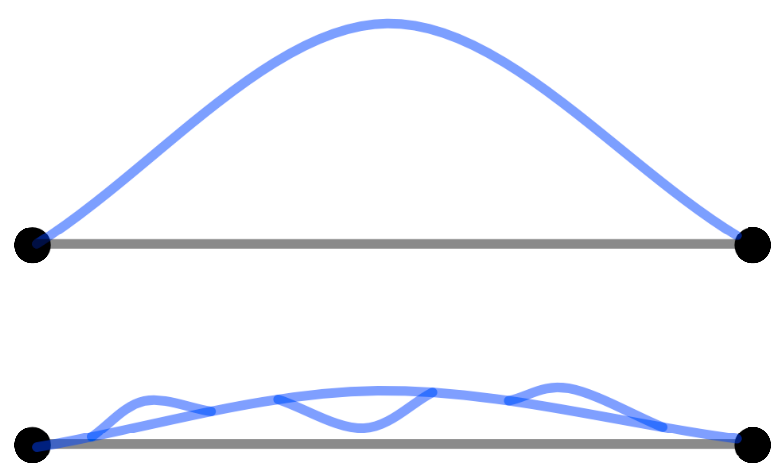
For small variations of A^{ν}, we can describe the variation of g^{+} as a rotation. The linearized rotational component of $A^{\nu,\mu}$ is $F^{\mu\nu} = A^{\nu,\mu} - A^{\mu,\nu}$.



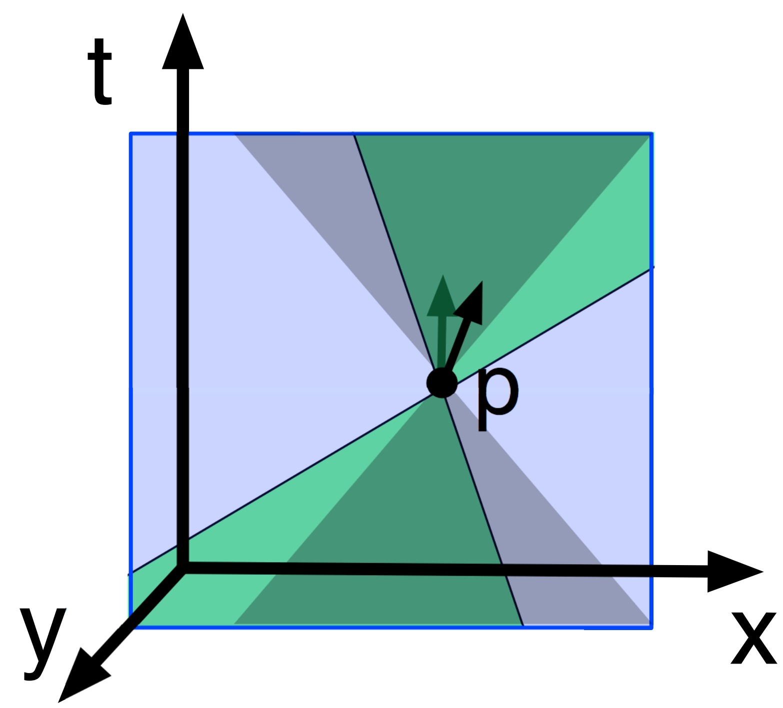
For weak fields, the effect on entropy is

$$L = (1/2)wF^{\mu\nu}F_{\mu\nu}$$

Lagrangian



Geometry: $-wR$



Field : $(1/2)wF^{\mu\nu}F_{\mu\nu}$

The total Lagrangian is

$$L = w((1/2)F^{\mu\nu}F_{\mu\nu} - R)$$

Interactions

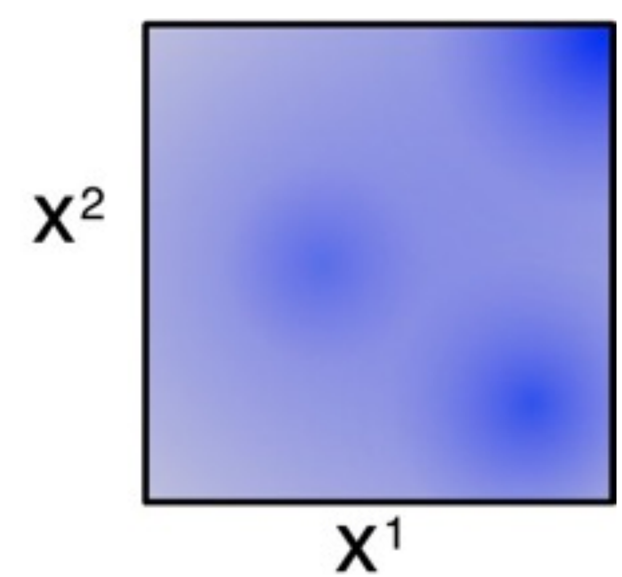
Gravity

Entropy is reduced by matter and energy, with Lagrangian wL_m, where L_m is the Lagrangian term of matter and energy.

The Lagrangian of curvature is $L = -wR$

Considering energy and curvature, $L = wL_m - wR$
If w is constant, $L = L_m - R$ as in general relativity.

Non-constant w on large distance scales may explain dark matter.

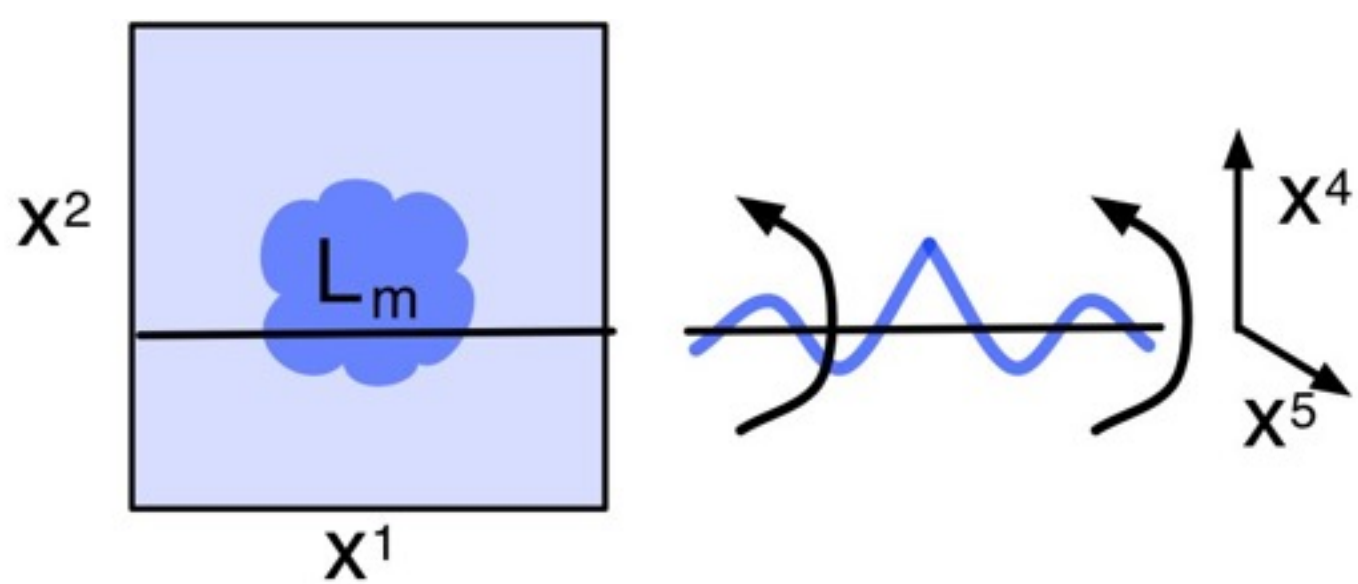


Geometry of gravity

Entropy is reduced by matter and energy, relative to the vacuum, by Lagrangian term wL_m.

wL_m can be partially offset by reduction of proper time: rotation in the co-dimension (x^4 and x^5).

The Lagrangian is maximized when everything rotates the same way. This is spontaneous parity breaking.

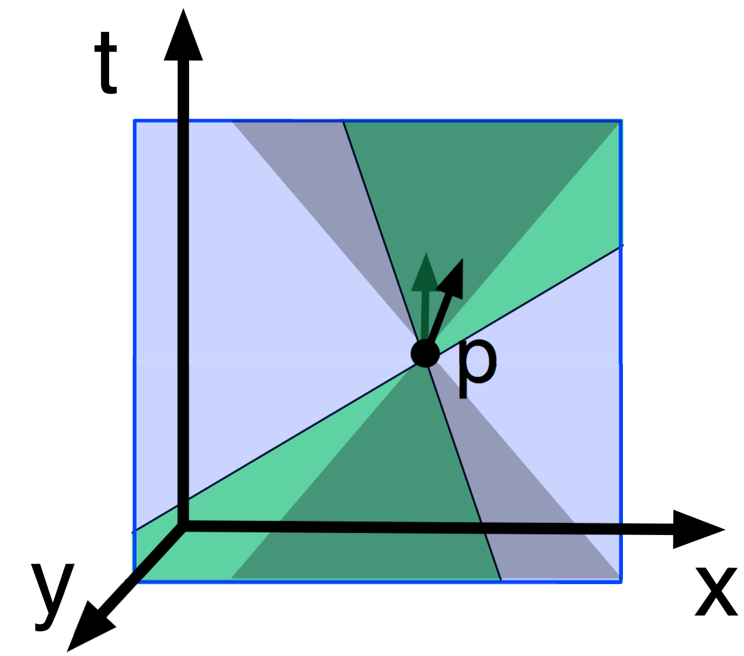


Electromagnetism

For constant w the Lagrangian

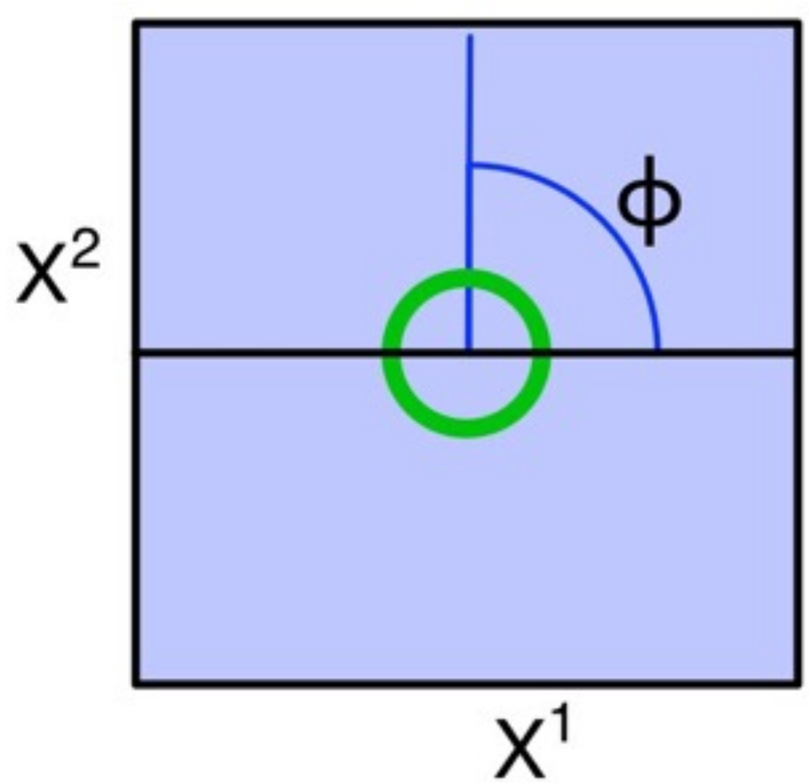
$$L = (1/2) w F^{\mu\nu} F_{\mu\nu}$$

is equivalent to Maxwell's equations.

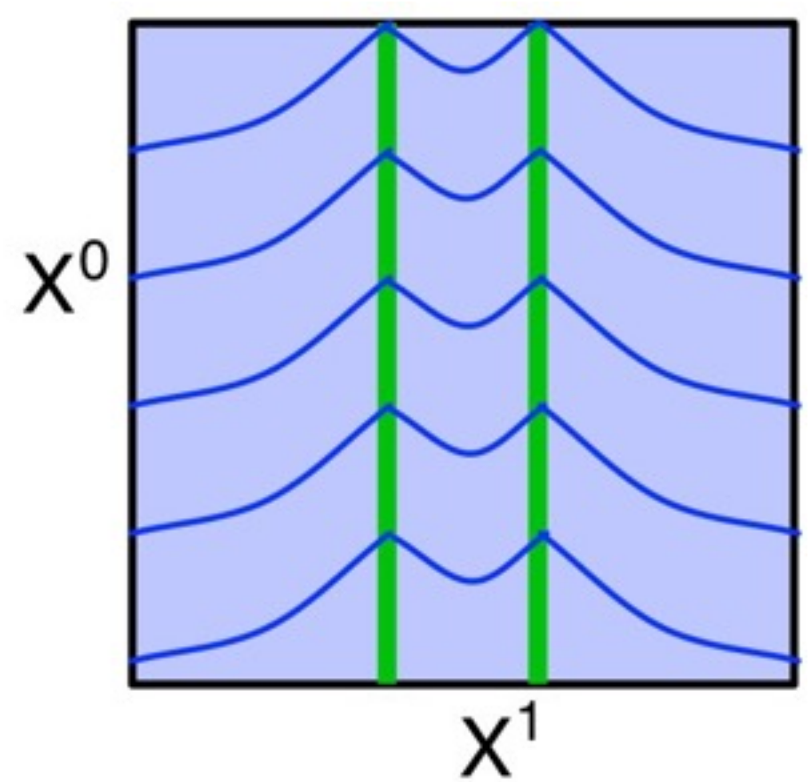


Charge

A charged particle is $\mathbb{R}^3\#(S^1 \times P^2)$ with A^ν field cusp.



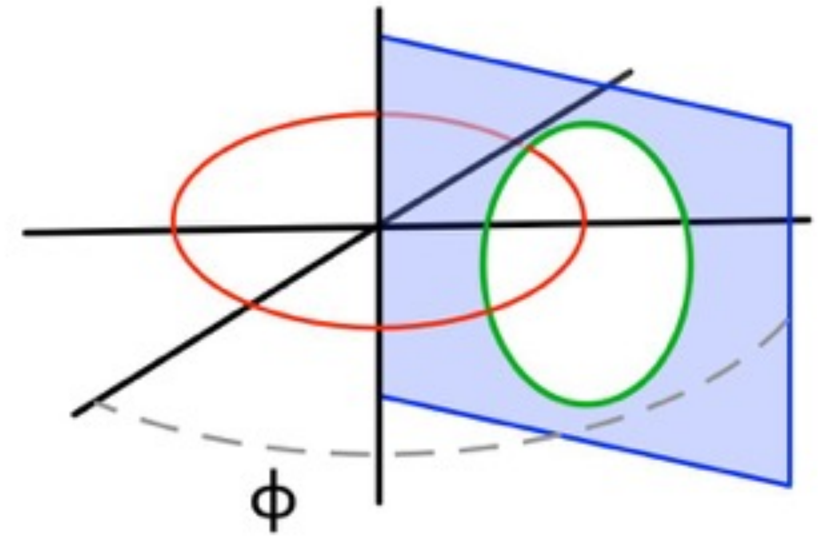
A slice through a charged $\mathbb{R}^3\#(S^1 \times P^2)$



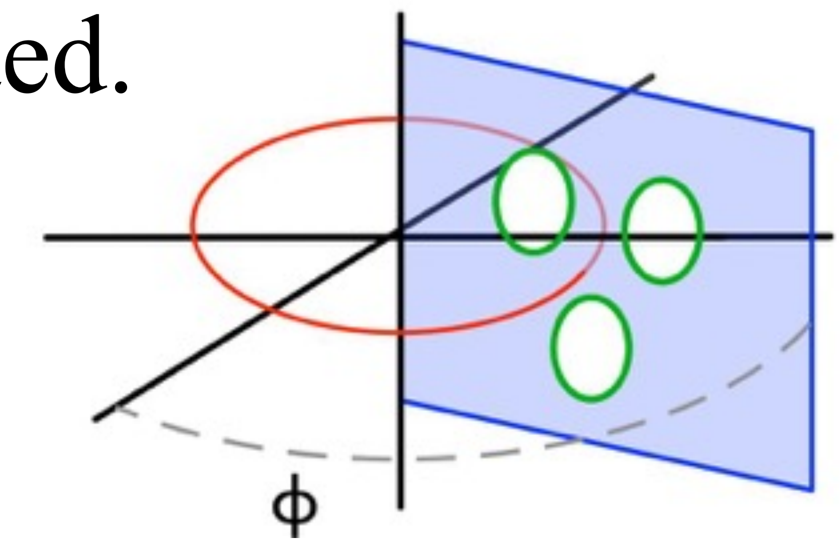
Lines of constant A^0 in that slice

Strong force

The topology $\mathbb{R}^3\#(S^1\times P^2)$ can link.
Linked $\mathbb{R}^3\#(S^1\times P^2)$ are quarks.



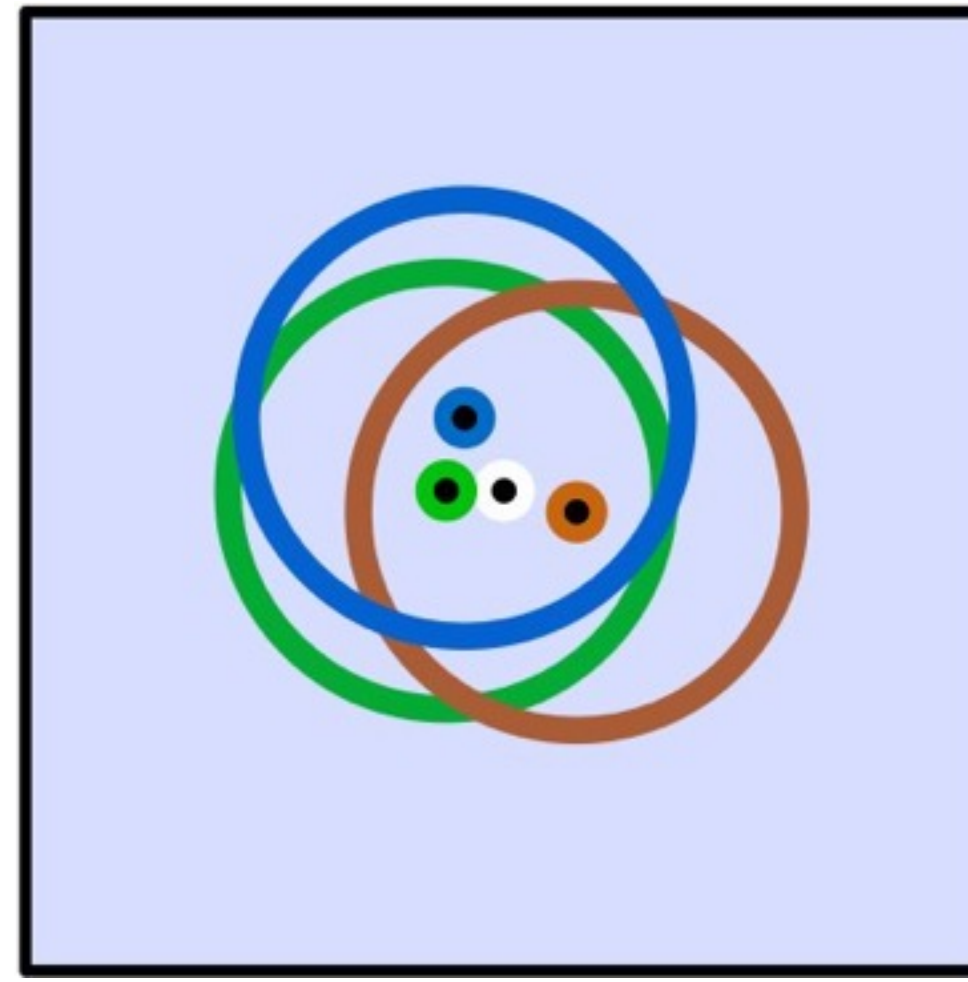
Linked $\mathbb{R}^3\#(S^1\times P^2)$ cannot be separated.
This is confinement.



At close distance, the links exert no force. This is asymptotic freedom.

Link $\mathbb{R}^3\#(S^1\times P^2)$ by linking $\mathbb{R}^2\#P^2$ then fibering over a circle.

Strong force: QCD



The center of each quark is displaced from the center of the particle by a 5-vector q_j. The 5-vectors sum to zero. To represent linking,

$$\left|q_j\right| = \left|(q_1, q_2, q_3, q_4, q_5)\right| \le 1$$

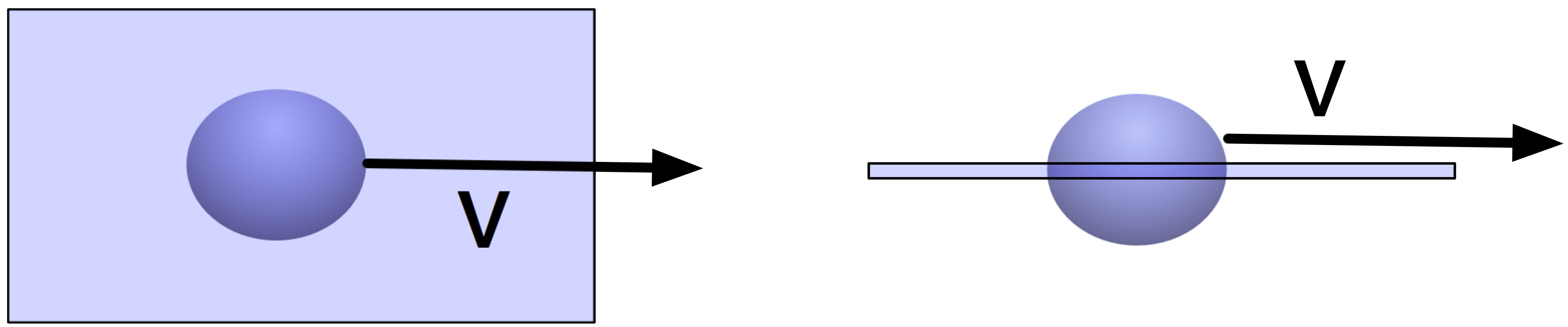
For some q_6 this is

$$\left|q_j\right| = \left|(q_1, q_2, q_3, q_4, q_5, q_6)\right| = 1 \Rightarrow$$

$$\left|q_j\right| = \left|(q_1 + q_2 i, q_3 + q_4 i, q_5 + q_6 i)\right| = 1$$ Color charge.

Gauge position with $SU(3)$.

Electroweak

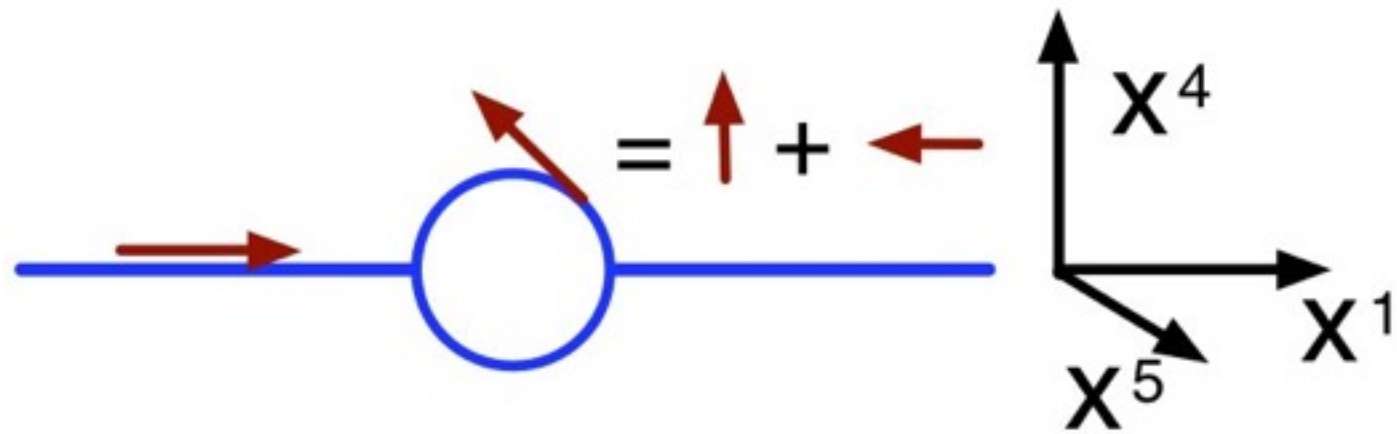


Near particles (knots), velocity may not be in the tangent space of M, which implies $F^{\mu\nu}$ field energy Lorentz transforms with rest mass.

If particles collide, they jiggle: virtual Higgs.

If they collide harder, they propagate a wave: real Higgs.

Electroweak: dimensions



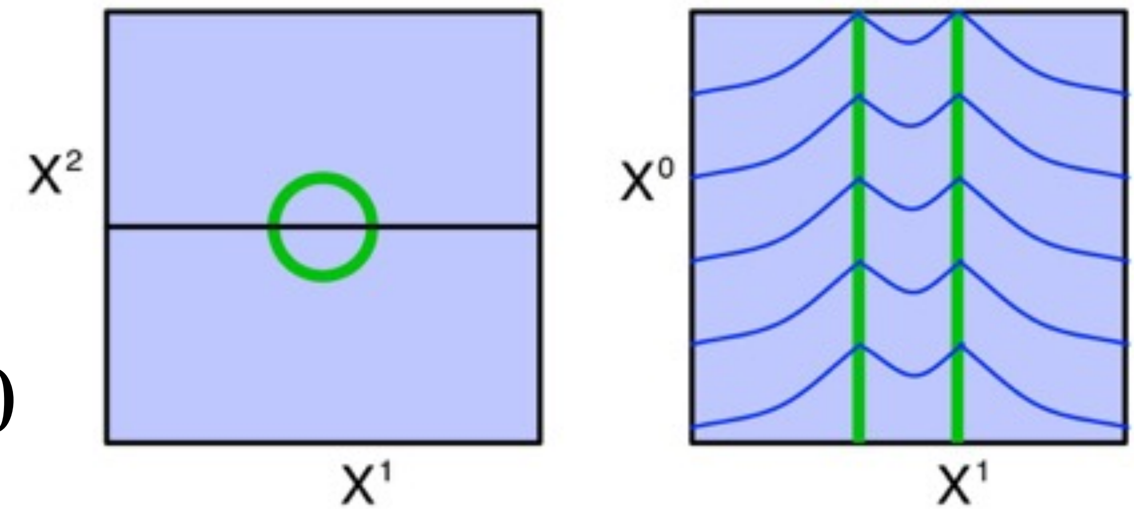
If the manifold is flat, rotation of $x^4\ x^5$ with gauge group $SO(2) \cong U(1)$ does not affect the manifold.

Close to knots, the manifold is not flat. Some tangent space components are unaffected by $SO(2)$ on $x^4\ x^5$. Other components are unaffected by $SO(3)$ on $x^1\ x^2\ x^3$. But fermions are non-orientable. Therefore use the double cover of $SO(3)$, which is $SU(2)$. The total gauge group is $SU(2) \times U(1)$.

Numerical application

Fine structure constant

A charged $\mathbb{R}^3 \#(S^1 \times P^2)$ has EM field and cusp.



But $g_{\mu\nu} = \rho^2 A^{\alpha}_{,\mu} A_{\alpha,\nu}$ and $\hat{R}^{\mu\nu} = 0$

Counter EM cusp effect with geometric field and cusp. EM field and geometric field together have spin angular momentum S.

$S(q^2) = \dfrac{\hbar}{2}$ therefore $\alpha^{-1} = \dfrac{4\pi\hbar}{q^2} = \dfrac{8\pi S(q^2)}{q^2}$

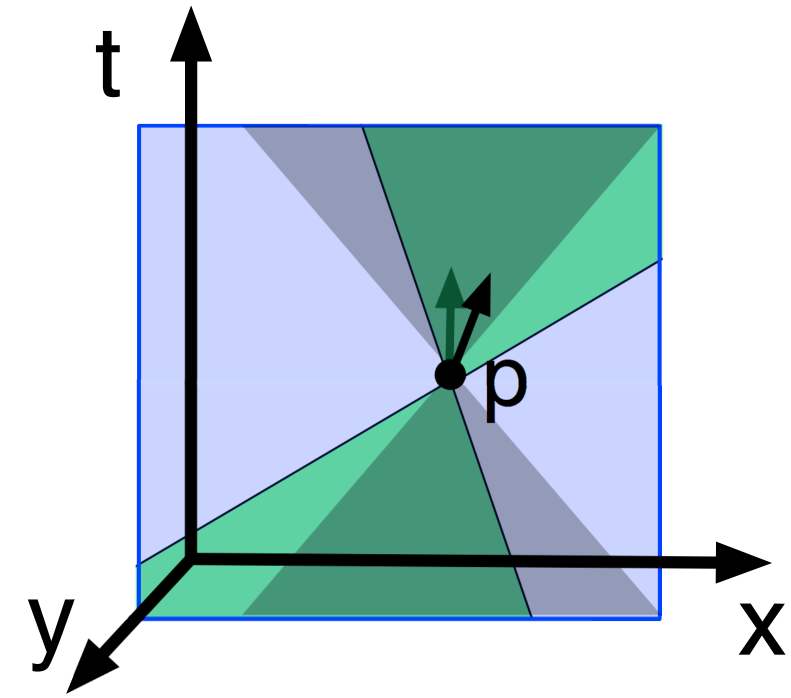
$\alpha_{est}^{-1} \approx 136.85$ error 0.13% ($\alpha_{\exp}^{-1} \approx 137.04$ actual)

Fine structure constant

The electromagnetic field is constrained by g^{+}.

The linearized Lagrangian is

$$L = (1/2) w F^{\mu\nu} F_{\mu\nu}$$



But the full Lagrangian is non-linear and the coupling constant of electromagnetism varies with field strength.

Summary

M is a branched 3+1-manifold.
M is embedded in a Minkowski 5+1-space.
The metric $g_{\mu\nu} = \rho^2 A^{\alpha}{}_{,\mu} A_{\alpha,\nu}$ is constrained.

Topology change produces knots $\mathbb{R}^3 \#(S^1 \times P^2)$.

Interaction of branches produces quantum interference.

Maximization of entropy produces fields and forces.

Further information

www.knotphysics.net

Physics on a Branched Knotted Spacetime Manifold
Knot physics: Neutrino helicity
Knot physics: Deriving the fine structure constant

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