# Knot Physics: Dark Matter

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## Abstract

We describe dark matter in knot physics. Knot physics assumes that spacetime is a branched 4-manifold embedded in a Minkowski 6-space. The theory has three metrics. The Minkowski space has the standard Minkowski metric $\eta_{\mu\nu}$. The second metric, $\bar{\eta}_{\mu\nu}$, is just the restriction of $\eta_{\mu\nu}$ to the spacetime manifold. In a previous work, we showed how mass and energy affect the curvature of $\bar{\eta}_{\mu\nu}$, reproducing results of general relativity. The third metric, $g_{\mu\nu}$, is used to constrain the branches of the spacetime manifold. In this paper, we derive an approximate relationship between $\bar{\eta}_{\mu\nu}$ and $g_{\mu\nu}$. The relationship implies $\bar{\eta}_{\mu\nu}$ can have non-zero Ricci curvature without a massive source particle. We show how this result has many of the characteristics of dark matter.


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## I. INTRODUCTION

Dark matter is a hypothetical form of matter whose existence is inferred from gravitational effects including the motion of visible matter, gravitational lensing, large scale structure of the universe, and the cosmological microwave background. Although these gravitational effects provide evidence for the existence of some type of dark matter, it is not known what is the precise form of dark matter, and direct detection has not been conclusively shown.

In a previous paper [1], we introduced the assumptions of knot physics and showed how those assumptions produce many of the results of the Standard Model. (For a complete list of those assumptions, see the Appendix of this paper.) In particular, we assumed that spacetime is a branched 4-manifold embedded in a Minkowski 6-space. The Minkowski space has the standard Minkowski metric $\eta_{\mu\nu}$. The spacetime manifold has the inherited metric $\bar{\eta}_{\mu\nu}$ that is the restriction of $\eta_{\mu\nu}$ to the manifold. (The inherited metric $\bar{\eta}_{\mu\nu}$ can be considered the shape of the spacetime manifold.) We also introduced a metric $g_{\mu\nu}$ and a fundamental constraint that $g_{\mu\nu}$ must be Ricci flat.

In the previous paper, we derived a Lagrangian and showed that matter and energy affect the metric $\bar{\eta}_{\mu\nu}$ the same way in both knot physics and general relativity. In other words, the shape of spacetime is affected by matter and energy in the same way in both general relativity and knot physics. The metric $g_{\mu\nu}$ is related to the metric $\bar{\eta}_{\mu\nu}$. Because of that relationship, Ricci flatness of $g_{\mu\nu}$ affects the shape of spacetime, $\bar{\eta}_{\mu\nu}$. In this paper, we show how that effect has the characteristics of dark matter.

## II. GRAVITY

In knot physics, there are three different metrics. There is a metric $\eta_{\mu\nu}$ that is the ordinary Minkowski metric, and it is defined everywhere on the Minkowski 6-space. There is a closely related metric $\bar{\eta}_{\mu\nu}$ that is the inherited metric on the spacetime manifold $M$. For any curve on $M$, the length of the curve relative to $\bar{\eta}_{\mu\nu}$ is the same as the length of that curve relative to $\eta_{\mu\nu}$, but the metric $\bar{\eta}_{\mu\nu}$ is defined only on $M$. The metric $\bar{\eta}_{\mu\nu}$ is the metric that is analogous to the metric of general relativity, it describes the shape of $M$ as an embedding in the Minkowski space.

In knot physics, the dynamics of the manifold are determined by maximization of entropy.

That entropy is approximated using a Lagrangian. In [1] we derive the Lagrangian

$$\mathcal{L} = w\Big(\frac{1}{2}F^{\mu\nu}F_{\mu\nu} - R\Big) \tag{1}$$

In this Lagrangian, we see the familiar terms of electromagnetism ($F^{\mu\nu}F_{\mu\nu}$) and gravity (scalar curvature $R$). The scalar curvature $R$ is taken with respect to the metric $\bar{\eta}_{\mu\nu}$, and it describes the curvature of $M$ as an embedding in the Minkowski space. At first glance, it appears that there are many terms from the Standard Model Lagrangian that are not represented in eqn.(1), but in [1] we showed how the geometry of a particle knot combines with the Lagrangian to produce all of the fields and forces. We can summarize the effects of particle geometry using a Lagrangian of matter and energy, $\mathcal{L}_m$. If the branch weight $w$ is constant, then eqn.(1) implies Einstein's field equation

$$\mathcal{L} = \mathcal{L}_m - R \tag{2}$$

This Lagrangian relates the matter and energy Lagrangian $\mathcal{L}_m$ to the curvature of the manifold as an embedding, given by $R$, which is the curvature relative to the metric $\bar{\eta}_{\mu\nu}$.

The third metric of knot physics is $g_{\mu\nu}$, which we use to constrain the spacetime manifold and define branch weight. In the next section, we explain the relevance of this metric to dark matter.

## III. BRANCH WEIGHT

In [1] we make assumptions that allow us to define a metric $g_{\mu\nu}$ that is the central component of this theory. For a full description of the motivation for those assumptions and the relevance of that metric, please see that paper. For the purposes of this paper, we will simply cite the relevant assumptions. (A complete list of the assumptions can be found in the Appendix of this paper.)

We assume a scalar field $\rho$. We assume a vector field $A^\nu$ such that $\det(A_{\alpha,\mu}A^{\alpha}_{\ ,\nu}) = -1$. We then define a metric $g_{\mu\nu} = \rho^2 A_{\alpha,\mu}A^{\alpha}_{\ ,\nu}$. We define a branch weight $w = (-\det(g))^{-1/2}$, which implies $w = \rho^4$. We require that $g_{\mu\nu}$ is Ricci flat, which we write $\hat{R}^{\mu\nu} = 0$.

From [1] we see that the field $A^\nu$ is equal to the electromagnetic potential $A^\nu_{EM}$ after a gauge transformation, $A^\nu = A^\nu_{EM} + x^\nu$. In the case that the electromagnetic field is small in comparison to the length scales being considered, we have the approximation $A^\nu_{EM} \approx 0$, which implies $A^\nu \approx x^\nu$, which implies $A_{\alpha,\mu} A^\alpha_{\ ,\nu} \approx \bar{\eta}_{\mu\nu}$, and therefore $g_{\mu\nu} \approx \rho^2 \bar{\eta}_{\mu\nu}$. Using $w = \rho^4$, we have $g_{\mu\nu} \approx w^{1/2}\, \bar{\eta}_{\mu\nu}$.

By one of our fundamental assumptions, the metric $g_{\mu\nu}$ is Ricci flat, $\hat{R}_{\mu\nu} = 0$. That assumption constrains the shape of the spacetime manifold through the relationship $g_{\mu\nu} \approx w^{1/2}\, \bar{\eta}_{\mu\nu}$. If there is matter and energy, then the Lagrangian implies gravity, which implies $\bar{\eta}_{\mu\nu}$ has non-zero Ricci curvature, $R^{\mu\nu} \neq 0$. We still have the constraint that $g_{\mu\nu}$ is Ricci flat, $\hat{R}^{\mu\nu} = 0$. With different curvatures, these two metrics can only satisfy our approximation $g_{\mu\nu} \approx w^{1/2}\, \bar{\eta}_{\mu\nu}$ if $w$ is not constant. This happens naturally, because particles already require that $w$ is not constant [1]. The gravitational contribution to scalar curvature of $\bar{\eta}_{\mu\nu}$ has some relatively small effect on the branch weight $w$ close to a particle.

On the other hand, assume that $w$ is not constant over large distance scales, as a result of some initial conditions of the spacetime manifold. In this case, the effect of $w$ may be more noticeable. We have the relationship $g_{\mu\nu} \approx w^{1/2}\, \bar{\eta}_{\mu\nu}$ and $g_{\mu\nu}$ is Ricci flat. If $w$ is not constant, then $\bar{\eta}_{\mu\nu}$ is forced to adjust. By assumption, we have $\hat{R}^{\mu\nu} = 0$, but non-constant $w$ may force $\bar{\eta}_{\mu\nu}$ to not be Ricci flat, $R^{\mu\nu} \neq 0$. In this case, the spacetime manifold does not satisfy Einstein's field equation for the vacuum. Instead, the geometry of spacetime appears to have a gravitational source, but without any corresponding particle. Variation of the conformal weight $w$

Matter has attractive gravitational self-interaction but branch weight does not. The gravitational self-interaction of matter is a consequence of maximizing the entropy of the manifold, and can be derived from the Lagrangian. For branch weight, maximization of entropy implies diffusion of the branch weight towards uniformity. In the vacuum, the branches of the manifold have the greatest number of possibilities for recombination when the branch weight is uniformly distributed.

The behavior of branch weight is slightly different if matter and energy are present. In this case, the matter and energy reduce the entropy of the spacetime manifold. We see from the Lagrangian, eqn. (1), that entropy is maximized when the branch weight $w$ is low in the presence of energy and high elsewhere. In that optimal arrangement, the matter reduces the entropy of the manifold, but it does so on fewer branches. If we assume a massive object, such as a galaxy, that has been equilibrating over a long time, we would expect that the branch weight near the galaxy would be lower than some asymptotic value farther from the galaxy. That condition of lower branch weight in a compact region implies that the metric $\bar{\eta}_{\mu\nu}$ must have positive scalar curvature $R$ in order to preserve Ricci flatness for the metric $g_{\mu\nu} \approx w^{1/2}\,\bar{\eta}_{\mu\nu}$. Thus we see that a massive object, such as a galaxy, is capable of creating its own dark matter if it has a long time to equilibrate. This interaction between branch weight and matter occurs on large distance scales. On small distance scales, the branch weight behavior is dominated by its self-interaction, which is diffusion. Combining these effects, we expect to see massive objects surrounded by dark matter halos, but those halos would be relatively smooth on short distance scales. From astronomical observation, we see that profiles of dark matter halos tend to be less cuspy than one would expect from cold dark matter particles [2]. The branch weight description of dark matter may provide a good fit for the dark matter halo of galaxies. See Fig. 1.

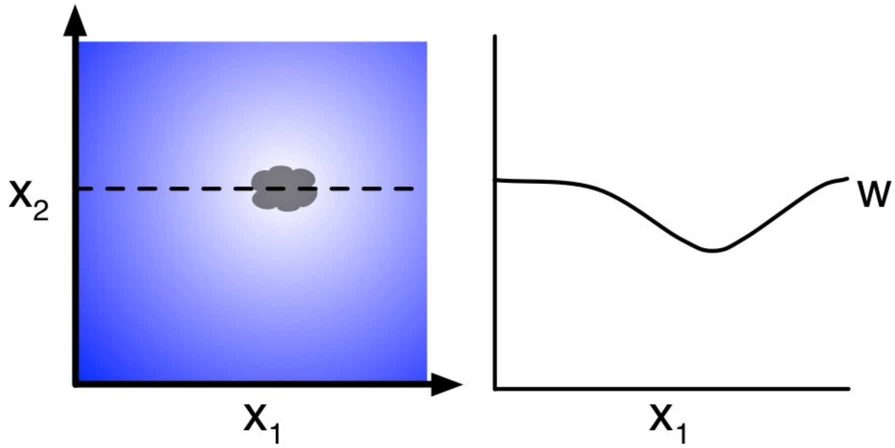


**FIG. 1** – In the left diagram we see a gray region of mass and energy, for example a galaxy. The surrounding space is colored blue with saturation of the color indicating the branch weight. Over time, the entropy of spacetime in this region is maximized by reducing the amount of branch weight in the area around the mass. We take a slice through the manifold and show, in the diagram on the right, the branch weight as a function of $x_1$. It is shown here without a cusp. This type of dark matter halo profile may be more consistent with the dynamics of branch weight than cold dark matter.

## V. COSMOLOGICAL CURVATURE

Astronomical observation indicates that the large scale geometry of the universe is flat to within experimental error (WMAP, BOOMERanG, and Planck for example). In knot physics, the large scale curvature is constrained by the fundamental assumption $\hat{R}^{\mu\nu} = 0$ that constrains the curvature of $g_{\mu\nu}$. The branch weight $w$ varies as a function of position, with some locations having a larger than average amount and other places having less. The relationship $g_{\mu\nu} \approx w^{1/2}\,\bar{\eta}_{\mu\nu}$ ensures that the average curvature of $\bar{\eta}_{\mu\nu}$ of the entire spacetime manifold is flat, in accordance with astronomical data.

## VI. CONCLUSION

In this paper we showed how the assumptions of knot physics produce results that are qualitatively similar to dark matter. The branched spacetime manifold is constrained by a few fundamental assumptions. The assumptions underconstrain the manifold and the manifold maximizes entropy. As we showed in a previous paper [1], one of the consequences of entropy maximization is that mass and energy affect the curvature of the metric $\bar{\eta}_{\mu\nu}$, in a way that matches general relativity. There is also a more fundamental constraint that affects the curvature of the spacetime manifold. Specifically, the metric $g_{\mu\nu}$ is Ricci flat. The approximate relationship $g_{\mu\nu} \approx w^{1/2}\,\bar{\eta}_{\mu\nu}$ implies that variation of the branch weight $w$ will affect the curvature of the metric $\bar{\eta}_{\mu\nu}$. We therefore see that variation of $w$ is the dark matter of this theory, because it produces spacetime curvature without any massive particle. We showed that the dark matter of this theory will tend to be found in proximity with ordinary matter and that it will tend to be smooth on small scales (rather than cuspy). These characteristics match astronomical observations of dark matter.

## VII. APPENDIX

### A. Assumptions

Knot physics is a unification theory that assumes spacetime is a branched manifold embedded in a Minkowski space. The theory is described in [1], and that description will be helpful for a more complete understanding of this paper. We list the assumptions of knot physics here for reference.

- **We assume a Minkowski 6-space $\Omega$.** The metric on $\Omega$ is $\eta_{\mu\nu} = \mathrm{diag}(1, -1, -1, -1, -1, -1)$. The coordinates are $x^\nu$.

- **We assume a branched 4-manifold $M$ embedded in $\Omega$.** A *branch* of $M$ is any closed unbranched 4-manifold $B$ without boundary that is contained in $M$. The metric $\bar{\eta}_{\mu\nu}$ on $M$ is inherited from $\Omega$. For convenience of coordinates we assume that, if $M$ is flat, then $M$ is in the subspace spanned by $x^0, x^1, x^2, x^3$.

- **We assume non-self-intersection of each branch of $M$.** For any branch $B$, the branch $B$ cannot intersect itself. This is necessary to prevent knots from spontaneously untying.

- **We assume a vector field $A^\nu$.** The field satisfies $\det(A_{\alpha,\mu}A^{\alpha}{}_{,\nu}) = -1$.

- **We assume a conformal weight $\rho$.** Then we define the metric $g_{\mu\nu} = \rho^2 A_{\alpha,\mu} A^{\alpha}_{\ ,\nu}$ and a Ricci curvature $\hat{R}^{\mu\nu}$ based on $g_{\mu\nu}$.
- **We assume a constraint on $g_{\mu\nu}$ relative to $\eta_{\mu\nu}$.** The metrics $g_{\mu\nu}$ and $\eta_{\mu\nu}$ define sets $g^+$ and $\eta^+$, and we assume that $g^+$ must intersect $\eta^+$.
- **We assume Ricci flatness $\hat{R}^{\mu\nu} = 0$ for $g_{\mu\nu}$.**
- **We assume that the weight $w = (-\det(g))^{1/2} = \rho^4$ is conserved at branching.**
- **We assume a lower limit $w \geq 1$.** This implies that the manifold can branch only a finite number of times.

### B. An overview of branch weight

In knot physics, the spacetime manifold is branched, and quantum mechanics is a consequence of the interactions of the branches. There is a metric, $g_{\mu\nu}$, that is the central component of the theory. Among other things, the metric $g_{\mu\nu}$ regulates the branching of the manifold. We obtain a branch weight $w = (-\det(g))^{1/2}$ and we require that this branch weight is conserved at branching. Whenever one branch splits into two, the sum of the branch weights must be conserved. We also require that $w \geq 1$ for every individual branch, and this implies that the branches can only split a finite number of times. This framework was motivated by quantum mechanics: quantum mechanics motivates branches, and branches motivate branch weight. In this paper, we see that the metric and its branch weight also lead to a natural explanation of dark matter. At a particular location, we can sum the branch weight of all the branches to get a total branch weight. The total branch weight can vary with position, and that variation affects the curvature of the spacetime manifold.

In Fig. 2 we take a slice through the spacetime manifold, showing how the total branch weight can vary with position. In Fig. 3 we take slices through two different examples of the spacetime manifold. In one example, the total branch weight is constant. In the other example, the total branch weight varies with position. In both examples, we show how the individual branches of the manifold depend on the total branch weight.

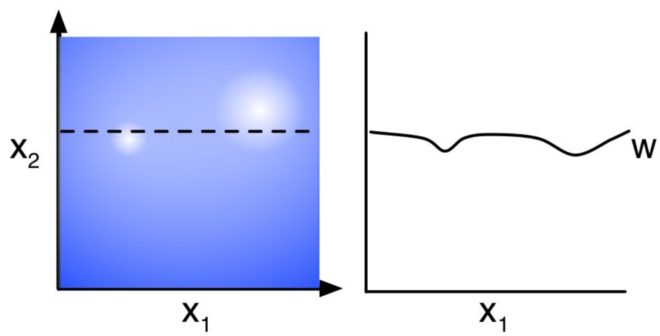


**FIG. 2** – We take a constant $x_2$ slice through spacetime, and plot the total branch weight $w$ as a function of $x_1$ on the right. The relationship $g_{\mu\nu} \approx w^{1/2}\,\bar{\eta}_{\mu\nu}$, implies that regions of lower branch weight $w$ have positive scalar curvature of the metric $\bar{\eta}_{\mu\nu}$, which implies dark matter.

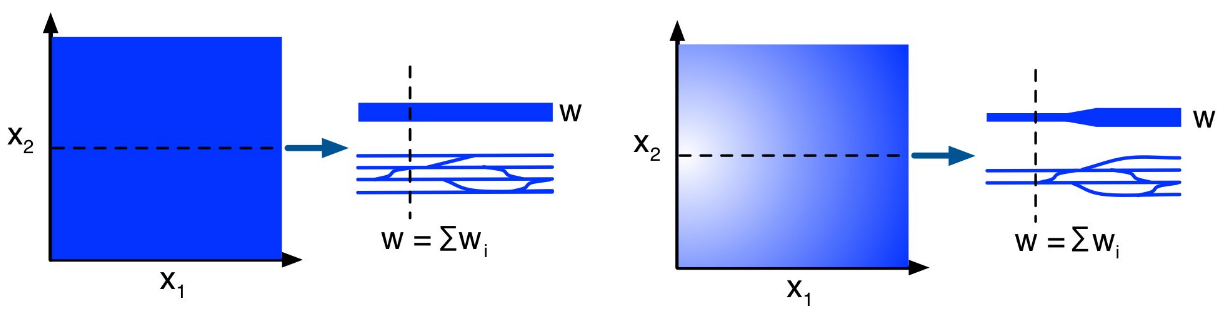


**FIG. 3** – We look at two different regions of spacetime. In the left diagram, the total branch weight $w$ is constant. We take a slice through the manifold and represent the total branch weight by the thickness of the blue line. (This is only a representation, the manifold itself has no actual thickness.) In knot physics, the spacetime manifold is branched, the branch weight is conserved when branches split, and the weight of each branch is constrained by $w_i \geq 1$. This implies that the total number of branches is bounded by the total branch weight. In the left example, we show the individual branches of the spacetime manifold, indicating that the sum of their individual branch weights is equal to the total, $w = \sum w_i$. Because each $w_i \geq 1$, this implies a bound on the number of branches. In the right diagram, the total branch weight increases going from left to right. We take a slice through the manifold, showing the branch weight in that slice. The width of the blue line increases, representing an increase of total branch weight. We again show the individual branches, which increase in number going from left to right.

### C. Curvature and branch weight

We have the relationship $g_{\mu\nu} \approx w^{1/2}\,\bar{\eta}_{\mu\nu}$, which we use to show that variation of $w$ forces curvature of $\bar{\eta}_{\mu\nu}$. We show here that variation of $w$ cannot be resolved with branching, and that curvature of $\bar{\eta}_{\mu\nu}$ is necessary. The proof is simple: branching changes the branch weight by a discrete amount at the location of branch separations. Curvature depends on derivatives of the weight $w$. There is no way to make a continuous function from a finite collection of step functions. For that reason, there is no way to resolve variation of branch weight $w$ with branching instead of curvature.

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[1] C. Ellgen **www.knotphysics.net** Physics on a Branched Knotted Spacetime Manifold

[2] G. Gentile, P. Salucci, U. Klein, D. Vergani, P. Kalberla The cored distribution of dark matter in spiral galaxies (2004) http://arxiv.org/abs/astro-ph/0403154